ECE 329

MEME STUDIES

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1 SUBJECT

70 Sheets

WIDE RULED

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CE 329 Fields and Waves I

ection X

Professor Minjoo (Larry) Lee (mllee@)

MWF 12-12:50 pm

2017 ECEB

ECE 329 - Fields and Waves I

Prof. Ilie . Waldrop

Office hours: W 3-4pm

See course webpage

lecture 1 - Monday, 28 August 2017

solar storms - geomagnetic fields

geodesic

Journey to the Stars Whoops Good Luck

Maxwell's eqns

Heaviside

telegrapher's eqn

electric force - attractive and repulsive

gravitational force 38 magnitudes weaker

weak/strong nuclear only attractive

- strong nuclear force keeps nucleus together

as $\frac{1}{r^2}$

- quantum forces keep electrons from falling into the nucleus

electron cloud

Coulomb's law

force $\propto \frac{1}{r^2}$

Lorentz force

$\vec{F} = q(\vec{E} + v \times \vec{B})$

ECE 350

Lorentz vs Lorenz

Electric and Magnetic Fields

Fields - measure of an interaction

Scalar and Vector Fields

gradient, curl, div

vector calculus & algebra

dot, cross products

ECE 329 W 30 Aug

Exams: 9/28, 10/19, ~~10~~ 11/16, Final 12/18

Flux - velocity, electric, magnetic flux of vector fields

Circulation

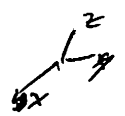
Vector notation, magnitude, Dot/Scalar/Inner Product

Commutative $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

Scalar Mult

Distributive $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

Check if (Assoc?)???

Cross Product  6×10^{-5} V/m·T

Electrostatics

Maxwell's Eqns (for Electrostatics)

$\nabla\times\vec{E} = 0$ Static case: NOTHING ~~changes~~ depends on time

$\nabla\cdot\vec{D} = \rho$ $\vec{E}$ and $\vec{B}$ are not interconnected

$\nabla\times\vec{H} = \vec{J}$ Helmholtz

$\nabla\cdot\vec{B} = 0$

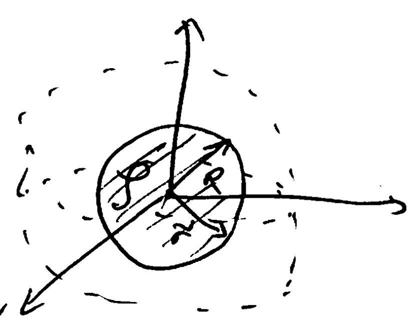
Coulomb's Law

Point charges

Principle of superposition

Gauss' law $Q = \oint_S \vec{D}\cdot d\vec{S}$

$Q_{encl.} = \oint \vec{D}\cdot d\vec{S} = Q = \int_V \rho\, dV$

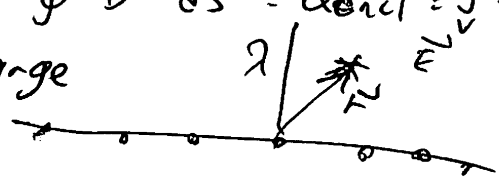


Ex spherical charge distribution of R

$r<R$ $\oint \vec{D}\cdot d\vec{S} = Q_{encl} = \int_V \rho\, dV$

$r>R$ $\oint \vec{D}\cdot d\vec{S} = Q_{encl} = \int_V \rho\, dV$

Line of charge



ECE 329 F 1 Sept.

1. LORENTZ FORCE

$\vec{F}_L = q(\vec{E} + \vec{v} \times \vec{B})$

2. COULOMB'S LAW

$\vec{F} = q\vec{E}$; $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r}$; valid only for electrostatics

3. GAUSS' LAW

$\oint_S \vec{D} \cdot d\vec{S} = Q_{encl.}$; valid for any surface shape, for any # of charges

$\oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{encl}}{\varepsilon_0} \rightsquigarrow \Phi_E = 0 \Rightarrow Q_{encl.} = 0$

$[\varepsilon_0] = \frac{F}{m}$ permittivity of free space

$\vec{D} = \varepsilon_0 \vec{E}$

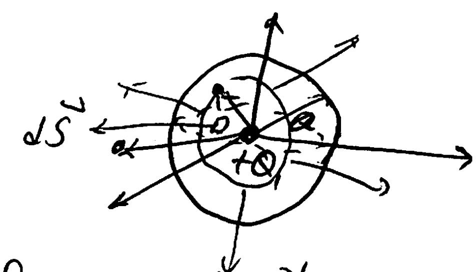
4. PRINCIPLE OF SUPERPOSITION

$Q_n, n > 1 \Rightarrow \vec{F}_{total} = \sum_{n=1}^{\infty} \frac{1}{4\pi\varepsilon_0} \frac{Q_n}{|r - r_n|^2} \frac{(\vec{r} - \vec{r}_n)}{|\vec{r} - \vec{r}_n|}$

Electrostatics = 2 & 4

Ex Spherical charge distribution distributed uniformly

$\vec{E}, \vec{D} = ?$ everywhere



1 $r \leq a$ $\Phi_E = \oint_S \vec{E} \cdot d\vec{S} = \oint_S E_r (\hat{r} \cdot \hat{r}) dS = E_r S = E_r 4\pi r^2$

Use Gauss' law: $\oint \vec{E}\, d\vec{S} = \frac{Q_{encl}}{\varepsilon_0}$

$Q_{encl.} = \int \rho \, dV = \rho \frac{4}{3}\pi r^3$

$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi a^3}$

$\Downarrow$

$E_r 4\pi r^2 = \frac{Q}{\varepsilon_0 a^3} r^3$

$\boxed{E_r = \frac{Q}{4\pi\varepsilon_0} \frac{r}{a^3}}$

$E \sim r$

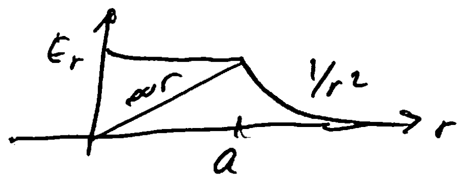
2 $r \geq a$: $\Phi_E = \oint_S \vec{E} \cdot d\vec{S} = E_r 4\pi r^2$

$Q_{encl} = Q \Rightarrow \boxed{E_r = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2}}$

$r \geq a$

$\Downarrow$

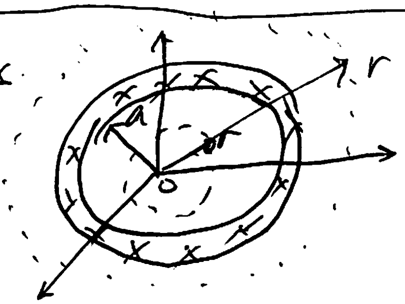
$E \sim \frac{1}{r^2}$



Gauss $$\oint_S \vec{E}\cdot d\vec{S} = \frac{Q_{encl}}{\epsilon_0}$$

$$\oint_S \vec{D}\cdot d\vec{S} = Q_{encl}$$

Ex



$\vec{E} = ?$

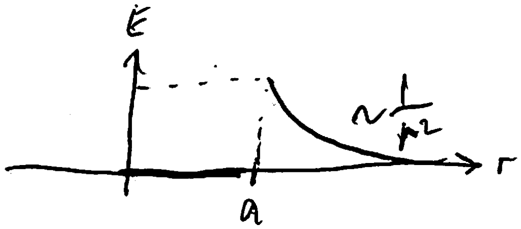
$r \le a \Rightarrow E_r 4\pi r^2 = \Phi_E$

$Q_{encl} = ? = 0$

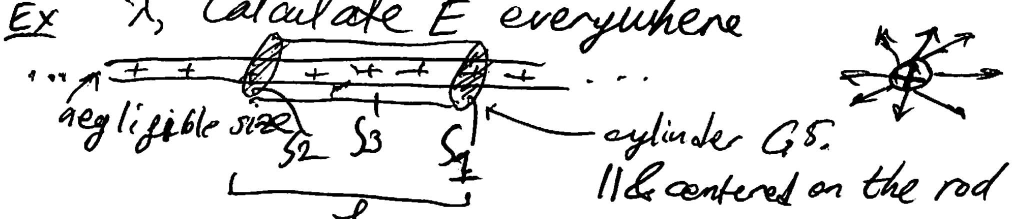
$\Rightarrow \boxed{E=0}$

$\Phi_E = \oint \vec{E}\cdot d\vec{S} = E\, 4\pi r^2$

$\Phi_E = \frac{Q}{\epsilon_0}$ $\Rightarrow \boxed{E = \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}}$

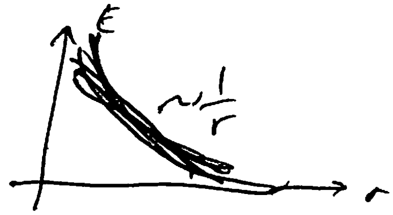


Ex λ, Calculate $\vec{E}$ everywhere



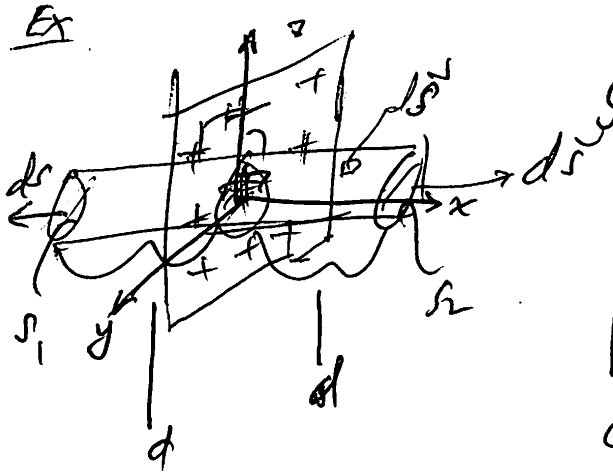
$$\oint \vec{E}\cdot d\vec{S} = \cancel{\oint \vec{E}\cdot d\vec{S}_1} + \cancel{\oint \vec{E}\cdot d\vec{S}_2} + \oint \vec{E}\cdot d\vec{S}_3$$

$$E = \frac{\lambda}{2\pi \epsilon_0} \frac{1}{r}$$



$Q_{encl} = \lambda l$

Ex



$\oint \vec{E} \cdot d\vec{S}$

σ, @ $x = 0$

$\vec{E}$ everywhere?

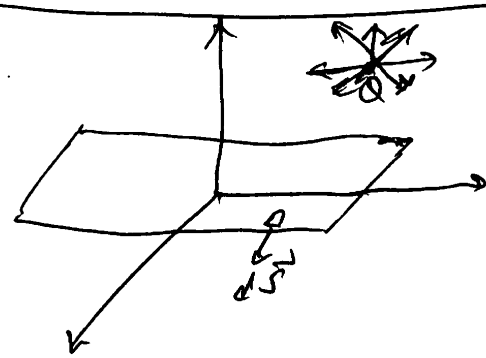
$$\boxed{E = \frac{\sigma}{2\epsilon}}$$

@ $r \ll$ size of slab $\sqrt{\text{Area}}$

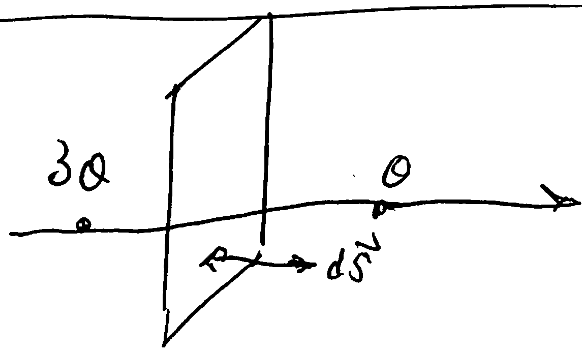
'infinity

$$\vec{E} = \hat{x} \frac{\sigma}{2\epsilon_0} \, \text{sgn}(x)$$

Ex



symmetry reasoning



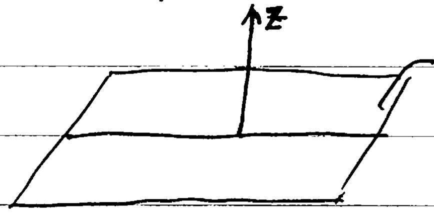
flux Q thru

Wed 6 Sept: Lecture 4

Objective: further Gauss' Law examples
introduce divergence and differential form of Gauss' Law

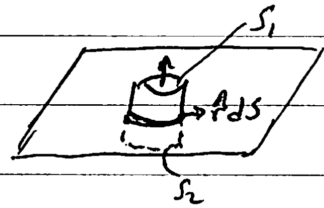
Ex Charged sheet

Gauss: $\oint_S \vec{D}\cdot d\vec{S} = \int_V \rho\, dV$



$\sigma = \left[\frac{C}{m^2}\right]$ $\quad \vec{D} = D_z\, \mathrm{sgn}(z)\, \hat{z}$

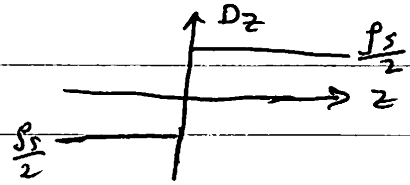
$$\mathrm{sgn}(z) = \begin{cases} 1 & z \geq 0 \\ -1 & z < 0 \end{cases}$$



area = A

$$\oint_{top} \vec{D_z}\,\hat{z}\cdot\hat{z}\, dS_1 + \oint_{bottom} D_z(-\hat{z})\cdot(-\hat{z})\, dS_2$$
$$= \int \rho\, dV = \rho_s A$$
$$= 2D_z A$$

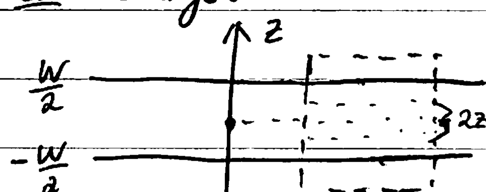
$$D_z = \rho_s/2$$



Suppose charged sheet moved to z = 5 m.

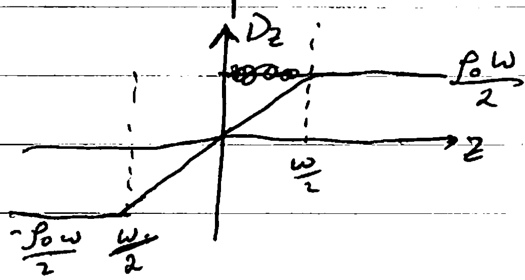
$$\vec{D} = D_z\, \mathrm{sgn}(z-5)\,\hat{z}$$

Ex Charged slab



$$\rho(z) = \begin{cases} \rho_0 & -\frac{W}{2} < z < \frac{W}{2} \quad (\text{and } \rho_0 > 0) \\ 0 & \text{elsewhere} \end{cases}$$

Outside slab $\quad D_z = \pm \frac{\rho_0 W}{2}$

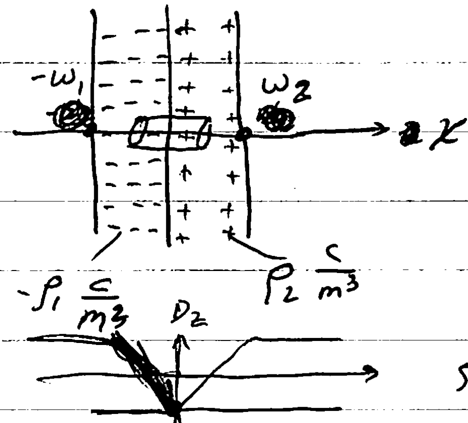


Inside slab $\quad D_z\, 2A = \rho_0 \cdot A \cdot 2z$

$$D_z = \pm \rho_0 z$$

<u>Ex</u> negative slab next to positive slab (pn junction diode) (ECE340)

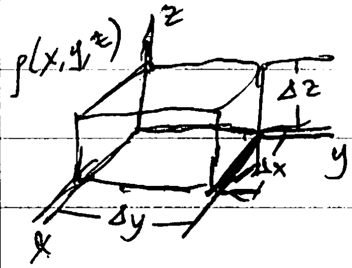
$w_1\rho_1 = w_2\rho_2$ (net neutral)



See solution in course notes.

Super impose two fields.

Differential form of Gauss's law



	surface	$d\vec{S}$	$\int \vec{D}\cdot d\vec{S}$
①	$x=x$	$-\hat{x}\,dy\,dz$	$-D_x\big\vert_{x=x}\Delta y\Delta z$
②	$x=x+\Delta x$	$\hat{x}\,dy\,dz$	$D_x\big\vert_{x=x+\Delta x}\Delta y\Delta z$

$$\oint \vec{D}\cdot d\vec{S} = \{D_x\big|_{x=x+\Delta x}$$

Divergence: measures the spatial variation of field magnitude <u>along</u> the direction of the field itself

→ ⟶ ⟶ ⟶ yes divergence

⟶ ⟶ ⟶ ⟶ no divergence

$$\nabla\cdot\vec{D} = \rho$$

Monday 11 Sept. Lecture 6

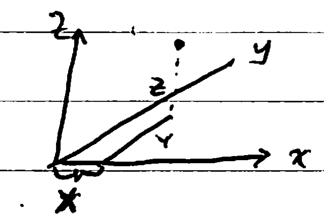
Example 3 from lecture 5

$V_0 = V(0,0,0) = 0$

$\vec{E} = 2x\hat{x} + 3z\hat{y} + 3(y+1)\hat{z} \quad \frac{V}{m}$ ← $\underbrace{\text{Curl-free field}}_{\text{path-independent}}$

Electrostatic field

$V_p = ?$ at $p = (X, Y, Z)$ in volts



$$V_p = \int_p^{\infty} \vec{E}\cdot d\vec{l} = -\int_0^p \vec{E}\cdot d\vec{l}$$

$$= -\int_0^X 2x\,dx \Big|_{y,z=0} - \int_0^Y 3z\,dy \Big|_{x=X,\,z=0}$$

$$- \int_0^Z 3(y+1)\,dz \Big|_{x=X,\,y=Y}$$

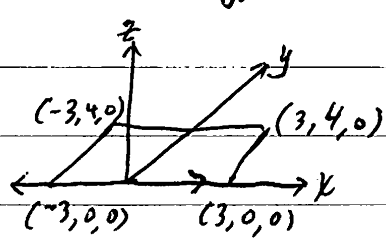
$$= -X^2 - 0 - 3(Y+1)Z$$

$$\boxed{V(x,y,z) = -x^2 - 3(y+1)z \quad V}$$

Circulation and boundary conditions

For curl-free fields and closed paths C, $\underbrace{\oint_C \vec{E}\cdot d\vec{l}}_{\text{"Circulation" of } \vec{E} \text{ over closed path } C} = 0$

Ex 1 $\vec{E}(x,y,z) = \hat{x}\,\frac{\rho x}{\epsilon_0}$ Show: $\oint \vec{E}\cdot d\vec{l} = 0$



$$\int_{x=-3}^{3} \overbrace{\hat{x}\frac{\rho x}{\epsilon_0}}^{\vec{E}} \cdot \overbrace{\hat{x}\,dx}^{d\vec{l}} + \int_{y=0}^{4} \hat{x}\frac{\rho 3}{\epsilon_0}\cdot\hat{y}\,dy + \int_{x=3}^{-3} \hat{x}\frac{\rho x}{\epsilon_0}\cdot\hat{x}\,dx$$

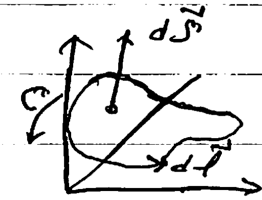
$$+ \int_{y=4}^{0} \hat{x}\frac{\rho(-3)}{\epsilon_0}\cdot\hat{y}\,dy$$

$$= \int_{-3}^{3} \frac{\rho x}{\epsilon_0}\,dx + \int_{3}^{-3} \frac{\rho x}{\epsilon_0}\,dx \; \boxed{= 0}$$

Divergence Theorem: flux per volume

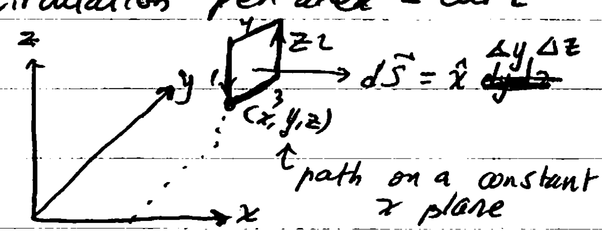
Stoke's Theorem: $\oint_C \vec{E}\cdot d\vec{l} = \int_S \nabla\times\vec{E}\cdot d\vec{S}$

circulation per unit area



curl fingers along C, $d\vec{S}$ is thumb

Circulation per area = curl



$$\oint_C \vec{E}\cdot d\vec{l} \approx E_z|_2\,\Delta z - E_y|_4\,\Delta y - E_z|_1\,\Delta z + E_y|_3\,\Delta y$$

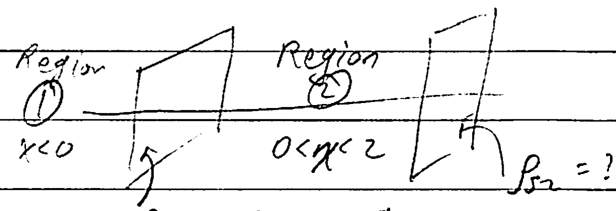
$$= (E_z|_2 - E_z|_1)\,\Delta z - (E_y|_4 - E_y|_3)\,\Delta y$$

$$= \hat{x}\,\Delta y\,\Delta z \cdot \nabla\times\vec{E}$$

$$\lim_{\Delta y,\Delta z\to 0} \frac{1}{\Delta y\,\Delta z}\oint_C \vec{E}\cdot d\vec{l} = \frac{E_z|_2 - E_z|_1}{\Delta y} - \frac{E_y|_4 - E_y|_3}{\Delta z} = \hat{x}\cdot\nabla\times\vec{E}$$

x component of $\nabla\times\vec{E}$ is circulation of $\vec{E}$, normalized by area on a constant x surface

Wed 13 Sept Lecture — Waldrop 11am



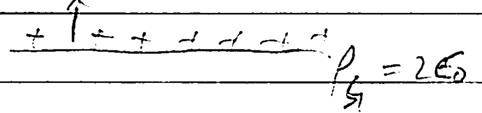
$\rho_{s_1} = 2\epsilon_0\,[C/m^2]$

given $\begin{cases} \rho_{s_1} = 2\epsilon_0\,[C/m^2] \\ \vec{E}_2 = 4\hat{x}+3\hat{y}+5\hat{z}\,[V/m] \\ E_{3x} = 2\,[V/m] \end{cases}$

ⓐ $\vec{E}_1 = ?$

$\vec{E}_{t_1} = \vec{E}_{t_2} = 3\hat{y}+5\hat{z}\quad [V/m]$

$\hat{n}=\hat{x}$



$D_{n2}-D_{n1} = 2\epsilon_0 = \epsilon_0 E_{n2} - \epsilon_0 E_{n1}$

$2\epsilon_0 = \epsilon_0(4) - \epsilon_0 E_{n1}$

$E_{n1} = 2\,[V/m]$

$\boxed{\vec{E}_1 = 2\hat{x}+3\hat{y}+5\hat{z}\,[V/m]}$

$\hat{n}\cdot(\vec{D}^+ - \vec{D}^-) = \rho_s$

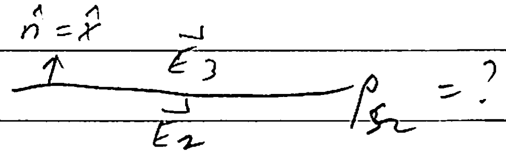
$D_n^+ - D_n^- = \rho_s$

ⓑ $\vec{E}_3 = ? = 2x \;\ldots$

ⓒ $\rho_{s_2} = ?$

$\hat{x}\cdot(\vec{D}_3 - \vec{D}_2) = \rho_{s_2}$

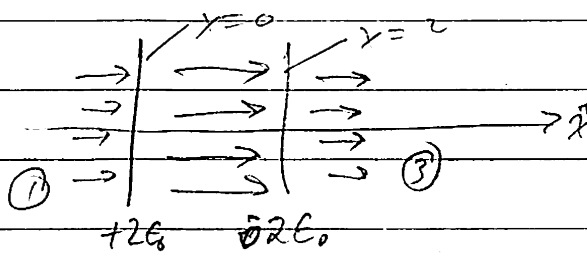
$\hat{n}=\hat{x}$



$\epsilon_0 E_{3x} - \epsilon_0 E_{2x} = \rho_{s_2}$

$\epsilon_0\,2 - \epsilon_0\,4 = \rho_{s_2} = -2\epsilon_0$

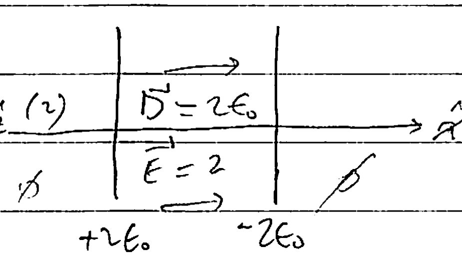
ⓓ $\vec{E}_{bkgd} = ?$



$\vec{E}_{sheets} = 2\hat{x}$ ②

$E_{total} = 4\hat{x}+3\hat{y}+5\hat{z}$ (2)

$\boxed{\vec{E}_{bkgd} = 2\hat{x}+3\hat{y}+5\hat{z}\,[V/m]}$



<u>Recall</u> $\nabla \cdot \vec{D} = \rho = \nabla \cdot (\epsilon_0 E)$

$\nabla \times \vec{E} = 0$ (static) substitute:

$\vec{E} = -\nabla V$

$\nabla \cdot (\epsilon_0 (-\nabla V)) = \rho$

$-\epsilon_0 \underbrace{(\nabla \cdot \nabla V)} = \rho$

$\nabla^2 V$ "Laplacian"

$\underbrace{\nabla \cdot \nabla V}$

$$\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right)$$

$$= \left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right) = \nabla^2 V \quad \underline{\text{scalar}}$$

$\boxed{\nabla^2 V = \frac{-\rho_0}{\epsilon_0}}$ Poisson's equation

$\boxed{\nabla^2 V = 0}$ Laplace's equation ($\rho = 0$ everywhere)

Consider:

$\rho(\vec{r}) = Q\,\delta(\vec{r}) \rightarrow V(\vec{r}) = \frac{Q}{4\pi\epsilon_0 |\vec{r}|} = \frac{Q}{4\pi\epsilon_0 r}$

$\delta(\vec{r}) \rightarrow$ $\boxed{\text{Poisson eqn}}$ $\rightarrow \frac{1}{4\pi\epsilon_0 r}$ impulse response

$\rho(\vec{r}) \rightarrow$ $\boxed{\text{Poisson eqn}}$ $\rightarrow \int \rho(\vec{r}\,') \frac{1}{4\pi\epsilon_0 |\vec{r} - \vec{r}\,'|} d^3\vec{r}\,'$

$\hookrightarrow \int \rho(\vec{r}\,')\, \delta(\vec{r} - \vec{r}\,')\, d^3 r'$

★ Exam favorite

Laplace's eqn

Consider Laplace eqn first.

$\rho(x,y,z) = 0 \Rightarrow \nabla^2 V = 0$

solution is linear in x, y, z

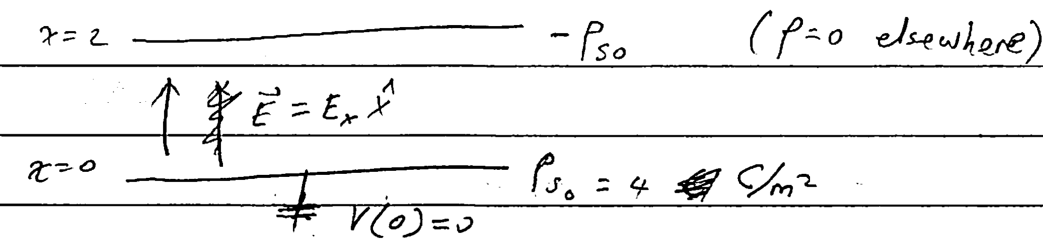
general soln $V(x) = Ax + B$

(1D) $V(x_1) = V_1$

$V(x_2) = V_2$

$-\nabla V(x) = \vec{E} = -A\hat{x}$

Ex



ⓐ T/F ? $V(x=2) > 0$ FALSE $V(2) < V(0)$

ⓑ $\boxed{V(2) = \,? = -8/\varepsilon_0 \text{ [V]}}$

$V(x) = Ax + B$ (since $\rho = 0$ inside)

$V(0) = 0 = B$

$V(x) = Ax = -E_x x$

$= -\frac{4}{\varepsilon_0} x$

$V(2) = -\frac{4}{\varepsilon_0}(2) = -\frac{8}{\varepsilon_0}$ [V]

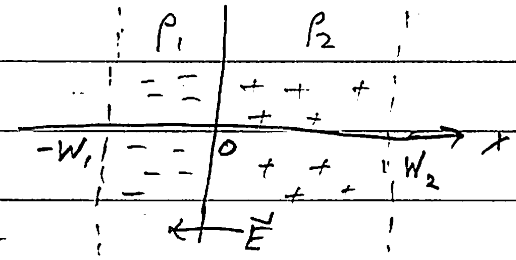
$\rho_s = 4$ —— $\uparrow \hat{x}$ $E_x\hat{x}$ $x=0$

ϕ

$\hat{x} \cdot (\varepsilon_0 E_x \hat{x} - 0) = 4$

$E_x = 4/\varepsilon_0$

Ex p-n junction semiconductor



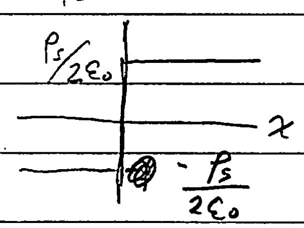
charge neutral: $\rho_1 W_1 = \rho_2 W_2$

overall question: $V(x) = ?$, $V(-W_1) - V(W_2) = ?$

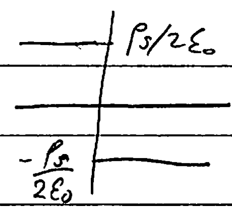
Recall:

Charged sheet

$\rho_s > 0$

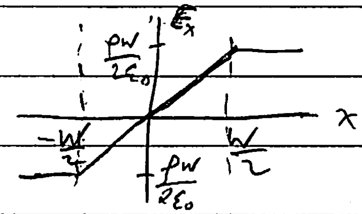


$\rho_s < 0$



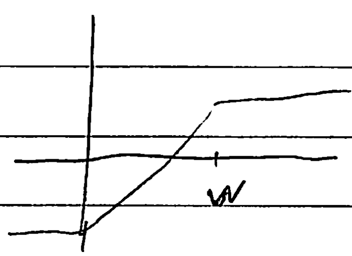
charged slab $\rho > 0$

@ $-W/2 < 0 < W/2$

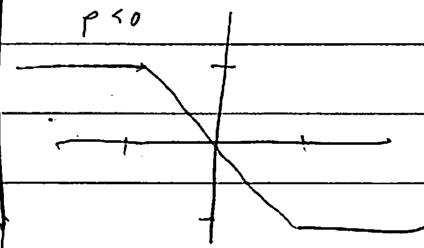


slab $\rho > 0$

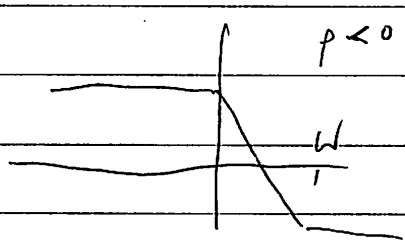
$0 < z < W$



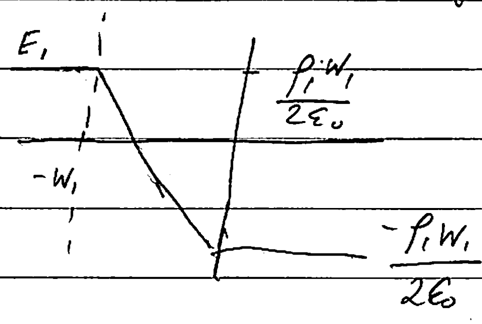
$\rho < 0$



$\rho < 0$



superposition of 2 charged slabs



$$\vec{E}_1 = \begin{cases} \dfrac{\rho_1 W}{2\epsilon_0} & x < -W_1 \\ \dfrac{\rho_1}{\epsilon_0}\left(x + \dfrac{W_1}{2}\right) & \\ -\dfrac{\rho_1 W_1}{2\epsilon_0} & x > 0 \end{cases}$$

Monday 18 Sept. Lecture 9 (Section X)

Static fields in dielectric media

$- \uparrow \epsilon_0 - -$

dielectric slab $\epsilon \neq \epsilon_0$

$\uparrow \epsilon_0$

$+ + + +$

Dielectric material

Non-conducting

(e.g., glass, bird poop)

a) $\vec{E} = 0$ metal/conductor

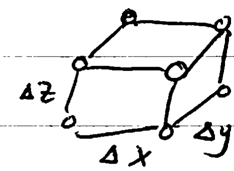
b) $\vec{E} > \vec{E}_0$

$\boxed{\text{c) } \vec{E} < \vec{E}_0}$

d) $\vec{E} = \vec{E}_0$ vacuum

Model: Array of dipoles (/lattice/crystals)

$$N_d = \frac{1}{\Delta x \Delta y \Delta z} \; m^{-3}$$



$$\rho_s = \frac{-e}{\Delta x \Delta y} \; \frac{C}{m^2}$$

$\Delta x, \Delta y$

$\Delta z \begin{cases} + \quad + \quad + \quad +\rho_s \\ - \quad - \quad - \quad -\rho_s \qquad \vec{E}_2 = 0 \\ + \quad + \quad + \end{cases}$

$- \quad - \quad - \qquad \vec{E}_1 = -\hat{z}\frac{\rho_s}{\epsilon_0} = -\hat{z}\frac{e/\epsilon_0}{\Delta x \Delta y}$

Look at space-weighted average to obtain macroscopic field

$$\vec{E}_p = \vec{E}_1 \frac{d}{\Delta x} + \underbrace{\cancel{\vec{E}_2}}_{0} \frac{\Delta z - d}{\Delta z}$$

$$= -\hat{z}\frac{ed/\epsilon_0}{\Delta x \Delta y \Delta z} = \frac{-N_d ed}{\epsilon_0}\hat{z} = \frac{-\vec{P}}{\epsilon_0}$$

$\vec{P} \equiv$ polarization field $\equiv N_d e d\hat{z} = N_d \vec{p}$

$\vec{p} = e\vec{d}$

Superposition

$\vec{E} = \vec{E}_0 + \vec{E}_p = \vec{E}_0 - \frac{\vec{P}}{\epsilon_0}$ ← Field is reduced inside dielectric

$\nabla \cdot (\epsilon_0 \vec{E}) = \nabla \cdot \epsilon_0 \vec{E}_0 - \vec{\nabla} \cdot \vec{P}$

$\rho = \epsilon_0 \nabla E_0$ (Gauss's Law)

$\nabla \cdot (\epsilon_0 \vec{E}) = \rho - \nabla \cdot \vec{P}$

$\nabla \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho \longrightarrow \vec{D} = \epsilon_0 \vec{E} + \vec{P}$

$\nabla \cdot \vec{D} = \rho$ ← Usual Gauss's law

Many materials have linear response

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$\chi_e \geq 0 \equiv$ electric susceptibility

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \underbrace{\epsilon_0 (1+\chi_e)}_{\epsilon} \vec{E}$$

$$\vec{D} = \epsilon \vec{E} \quad , (1+\chi_e) = \epsilon_r$$

	ϵ_r relative permittivity
vacuum	1
air	10000
glass	4-10

In perfect dielectrics $\nabla \cdot \vec{D} = 0$ since $\rho = 0$

Boundary separating perfect dielectrics $\{ \hat{n} \cdot (\vec{D}^+ - \vec{D}^-) = 0, \; D_n^+ = D_n^-$

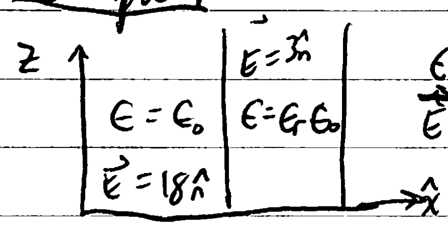
Also, $\hat{n} \times (\vec{E}^+ - \vec{E}^-) = 0, \; E_t^+ = E_t^-$

Interfaces b/t dielectric materials

Boundary between conductor/dielectric

$\hat{n} \cdot D^+ = \rho_s$ where $\hat{n}$ points from conductor to dielectric

Example 1



$\epsilon = \epsilon_0$
$\vec{E} = 18\hat{x}$

Solve: $\vec{D}, \vec{P}, \epsilon_r, \chi_e$

outside slab: $\vec{D}_{out} = \hat{x}\, 18\epsilon_0$

$\vec{P}_{out} = 0$

$\vec{D}_{slab} = \hat{x}\, 18\epsilon_0 = \epsilon_{slab} \vec{E}_{slab}$

$\vec{D}_{slab} = \epsilon_0 \vec{E}_{slab} + \vec{P}_{slab}$

$\epsilon_{slab} = 6\epsilon_0 \, , \; \epsilon_{slab} = 1 + \chi_e$

$\vec{P}_{slab} = (18\epsilon_0 - 3\epsilon_0)\hat{x} = \underline{\hat{x}\, 15\epsilon_0}$

$\chi_e = 5$

$\vec{D} = \epsilon \vec{E}, \; \epsilon = \epsilon_r \epsilon_0$

$$\nabla^2 V = -\frac{\rho}{\epsilon} \quad (\vec{E} = -\nabla V, \; \vec{\nabla} \cdot \vec{D} = \rho)$$

$$\vec{\nabla} \cdot \vec{D} = \nabla \cdot (\epsilon \vec{E}) = \epsilon \nabla \cdot \vec{E}$$

implied that $\epsilon_r \neq F(x, y, z)$

Sec. X

Wednesday 20 Sept Lecture 10

Objectives: wrap up perfect dielectrics, start capacitance and conductance

$\rho = 2\epsilon_0$ ——— $z=2$ $\boxed{V(2) = 0?}$

$2\epsilon_0$ - - - $z=1$

ϵ_0 ——— $z=0$

$\rho = 2\epsilon_0$ ⏚

Region $1<z<2$

$V(z) = A + B(z-1)$

Next use BCs to solve A, B

Region $0<z<1$

Laplace eqn $\nabla^2 V = 0$

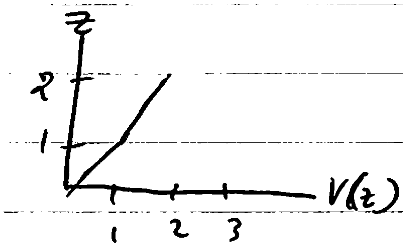
$V(z) = Az$ $\quad$ $@z=0$

$A = V(1)$ $\quad$ $\hat{z}\cdot\vec{D}(0) = \rho_s$

$\vec{E} = \nabla V = -A\hat{z}$ $\quad$ $-\epsilon_0 A = -2\epsilon_0$

$\vec{D} = \epsilon_0 \vec{E}$ $\quad$ $A = 2$

$$V(z) = \begin{cases} 2z & 0<z<1 \\ 2+(z-1) & 1<z<2 \end{cases}$$



Just below $z=1$

$\vec{D}(1^-) = 2\epsilon_0 \hat{z}$

Above $z=1$

$\vec{E} = -\nabla(A + B(z-1)) = -B\hat{z}$

$\vec{D}(1^+) = -2\epsilon_0 B\hat{z}$

Boundary condition

$\hat{z}(\vec{D}(1^+) - \vec{D}(1^-)) = 0$

$D_z(1^+) = D_z(1^-)$

$-2\epsilon_0 B = -2\epsilon_0$

$B = 1$

Example 4, Lec 9 notes

$\rho_s = -2\epsilon_0$ ——— $z=2$ $\quad$ $V(2) = ?$

$\epsilon(z) = \frac{4\epsilon_0}{4-z}$ (grades from ϵ_0 to $2\epsilon_0$)

$\rho_s = 2\epsilon_0$ ——— $z=0$

⏚ $V(0) = 0$

Cannot use Laplace's eqn due to inhomogeneity

Gauss' Law

$\nabla \cdot (\epsilon_0 \vec{E}) = \rho$

$\nabla \cdot (\epsilon \vec{E}) = 0$

$\frac{\partial}{\partial z}(\epsilon_0 E_z) = 0$

Product is constant

$\epsilon_z E_z(z) = \epsilon(0) E_z(0)$

$E_z(z) = \frac{\epsilon_0}{\epsilon(z)} E_z(0)$

$E_z(z) = E_z(0) \cdot (1 - \frac{z}{4})$

$\underline{BC \text{ at } z=0}$

$\hat{z} \cdot \vec{D}(0) = \rho_s$

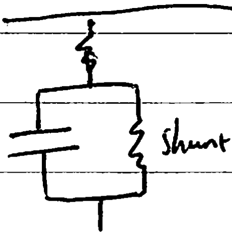
$D_z(0) = \epsilon_0 E_z(0) = 2\epsilon_0$

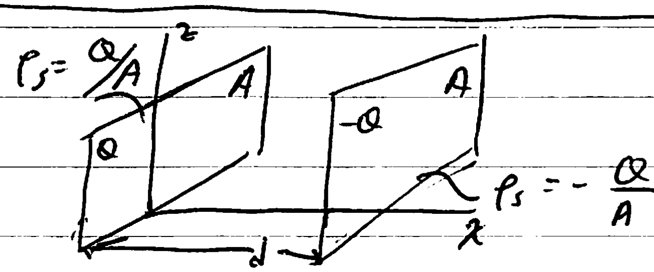
$E_z(0) = 2 \frac{V}{m}$

$E_z(z) = 2(1 - \frac{z}{4})$

$V(z=2) = \int_{z=2}^{0} \vec{E} \cdot d\vec{l}$

$= 2(z - \frac{z^2}{2})\Big|_2^0 = \boxed{-3V}$





fringing capacitance

For $d << \sqrt{A}$, plates "look" nearly infinite

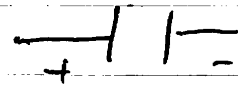
$\vec{D} = \hat{x}\frac{Q}{A}, \vec{E} = \frac{\vec{D}}{\epsilon_0} = \hat{x}\frac{Q}{\epsilon_0 A}$

$V = \int_{(0,0,0)}^{(d,0,0)} \vec{E} \cdot d\vec{l} = \int_0^d \hat{x}\frac{Q}{\epsilon_0 A} \cdot \hat{x}\, dx = \int_0^d \frac{Q}{\epsilon_0 A} dx$

$V = \frac{d}{\epsilon_0 A} Q$

$C = \epsilon_0 \frac{A}{d}$

Recall $Q = CV$



$I = C\frac{dV}{dt} = \frac{dQ}{dt}$

$\sqrt{A} \propto \lambda = c/f$ <u>Quasi-static assumption</u>

$f = 10^7 s^{-1}$ (10 MHz), $\lambda = 30m$

$P = VI$

$V \cdot C \frac{dV}{dt}$

$= \frac{d}{dt}\left(\frac{1}{2} C V^2\right)$

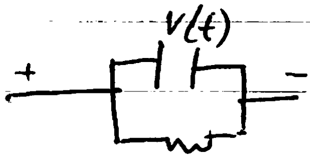
Stored energy

$\int P\,dt = \frac{1}{2} C V^2 = \frac{1}{2}\epsilon_0 |E_T|^2 \underset{\sim}{Ad}$

Normalize by volume

$W = \frac{1}{2}\epsilon_0 E_x^2$

energy density



$I = G \cdot V(t) + C\frac{dV}{dt}$

pick up 3×5 card

Conservative field → electrostatic field (?)

time dilation/ Lorenz contraction

antiparallel

Monday 25 Sept Lecture 12 Sect. X

MIT Physics Demo - Forces on a Current Carrying Wire

Objective wrap up polarization current

Introduce magnetic force and fields

Exam Wednesday night

→ review sessions on Wednesday

$$\vec{r} = -\frac{e}{m\omega_0^2}\vec{E} \rightarrow \text{If } \vec{E} = f(t)$$

$$\vec{v} = \frac{d\vec{r}}{dt} = -\frac{e}{m\omega_0^2}\frac{d\vec{E}}{dt}$$

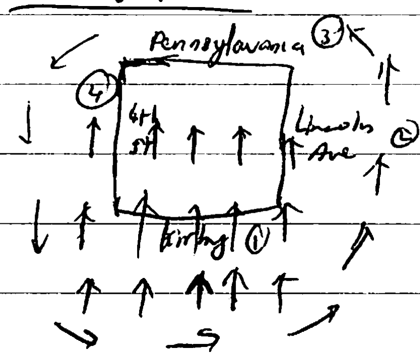
$$\vec{p} = \epsilon_0 \chi_e \vec{E} \qquad \vec{J}_p = -eN_d\vec{v} = \frac{N_d e^2}{m\omega_0^2}\frac{d\vec{E}}{dt} = \frac{d\vec{P}}{dt}$$

Dielectric : No DC current

But AC polarization current exists

Aside:

Circulation



Magnetic Fields

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$$

$\vec{E}' \neq 0$

Lab frame e^- frame (′) prime frame

e^- e^- $\vec{J}\ \frac{A}{m^2}$ electrons are stationary

Wire is charge neutral

$\lambda_+ \left[\frac{C}{m}\right] = -\lambda_-$

No electrostatic field

$I = v\lambda_+ = v|\lambda_-|$

Length contraction

$x' = \gamma(x - vt)$, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$\lambda_+' = \gamma\lambda_+$

$\lambda_- = \gamma\lambda_-'$

$\lambda' = \lambda_+' - \lambda_-'$

$\lambda' = \lambda_+\left(\frac{v^2}{c^2}\right) \neq 0$

macroscopically observable — due to long physical consts.
~~unproven Max~~

$$\vec{E}' = \frac{\lambda'}{2\pi r \epsilon_0}\hat{r}$$

$\vec{F}' = q\vec{E}'$ force on stationary test charge

$$\simeq \frac{q\lambda_+\left(\frac{v^2}{c^2}\right)}{2\pi r \epsilon_0}\hat{r} \quad \text{Recall } I = \lambda_+ v$$

$$\vec{F}' \simeq \frac{qI\frac{v}{c^2}}{2\pi r \epsilon_0}\hat{r} = q\vec{v}\times\vec{B} = \vec{F}$$

I along $\hat{z}$, $\vec{v}$ ~~moves~~ along $-\hat{z}$

$$\vec{F} = q\vec{v}\times\boxed{\frac{I}{2\pi r}\left(\frac{1}{\epsilon_0 c^2}\right)\hat{\phi}}$$

$\vec{B}$ magnetic flux density $\frac{Wb}{m^2}$

Magnetic permeability

$$\frac{1}{\epsilon_0 c^2} \equiv \mu_0 = 4\pi\times 10^{-7}\ \frac{H}{m}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

(sending a telegraph – wire acts as a wave guide as light moves – not electrons – when voltage turned on & off)

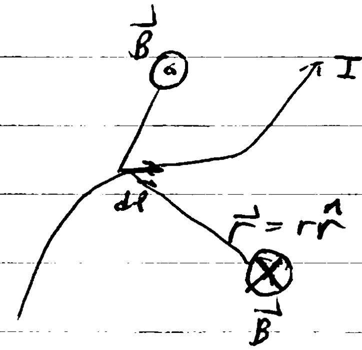
$\nabla\cdot\vec{B} = 0$

No divergence

for magnetostatics

Empirical law

("Unproven Maxwell's eqn")



$$d\vec{B} = \frac{\mu_0 I\, d\vec{l}\times\hat{r}}{4\pi r^2}$$

Biot-Savart Law

Ampere's Law

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed} = \mu_0\int \vec{J}\cdot d\vec{s}$$

circulation of a static magnetic field

flux over open surface

$\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$, $\nabla\cdot$, $D(\ )$, $D\times$, 3×3 det

Superposition potentials + fields

$dV = 4\pi r^2 dr$

Exam 1

Wed 27 Sept Lecture: Review Session

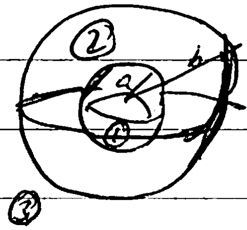
$V(b) = \frac{Q}{4\pi\varepsilon_0 r}$ [V]

Waldrop

SP 17 Q2:

$$\rho(r) = \begin{cases} \alpha r & a \le r \le b \\ 0 & \text{else} \end{cases}$$

total charge $= 0$



$$\oint_S \vec{D}\cdot d\vec{S} = Q_{encl} = \int_V \rho\, dV$$

$\hookrightarrow \vec{D} = \varepsilon_0 \vec{E}$

$\vec{E}_1 = ?$ 0

$\vec{E}_2 = ?$ $\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

$\vec{E}_3 = ?$ [V/m]

part ⓒ $\vec{E}_2 = ?$

$$\oint \vec{D}\cdot d\vec{S} = \oint D_r \underbrace{\overbrace{\hat{r}\cdot\hat{r}}^{1}\, ds}_{d\vec{s}} = D_r(r)\oint ds = D_r(r)\, 4\pi r^2$$

$$= Q_{encl}$$

$$\overset{3d}{=} \int_V \rho\, dV$$

$$= \int_a^r \rho\, 4\pi r^2\, dr$$

$$= \int_a^r \alpha r^3 4\pi\, dr = \frac{4\pi\alpha(r^4 - a^4)}{4}$$

Solve for

$$E_r(r) = \frac{D_r(r)}{\varepsilon_0} = \frac{1}{\varepsilon_0}\frac{(4\pi\alpha(r^4-a^4))}{4}\frac{1}{4\pi r^2}$$

ⓓ $V(r=b) = ?$ given $V(r=\infty) = 0$

$$V(b) - \underbrace{V(\infty)}_{0} = -\int_b^\infty \vec{E}\cdot d\vec{l} = -\int_b^\infty \underbrace{E_r}_{E_3\ (r \ge b)}\, dr$$

$$V(b) = \frac{Q}{4\pi\varepsilon_0 r}\ [V]$$

---

conductance / susceptability } prob not on exam

multi-slab dielectrics, potentials

SP 17. Q7 C

$E = -\nabla V$ $\quad \vec{D} = \varepsilon \vec{E}$ $\quad$ tangential, boundary cond,

parallel plate

$C = \epsilon_0 \frac{A}{d}$ $\qquad \vec{D} = \hat{x} \frac{Q}{A}$ $\quad Q = CV$

Sp 17 Q7

Give:

$z=3$ ——— $\leftarrow E=0$ for $z>3$

$\varepsilon_2 \quad V(z) = z - \frac{1}{6}$ $\qquad \rho_s = \pm 2\varepsilon_0 \; C/m^2$

$z=\frac{1}{2}$ $\varepsilon_1 \quad V(z) = \frac{2}{3} z$ $\qquad$ (don't know which is +/-)

$z=0$ ——— $\nwarrow E=0$ for $z<0$

a) $\left.\begin{array}{l} V(z=0) = 0 \\ V(z=3) = 3 - \frac{1}{6} > 0 \end{array}\right\}$ so $\vec{E}$ points $\downarrow -\hat{z}$ $\quad (+) \to (-)$

ANS: bottom plate has $-\rho_s$ since $V(0) < V(3)$

b) $\vec{E} = ? = -\nabla V = -\frac{\partial V}{\partial z}\hat{z} = \begin{cases} -2/3 \; \hat{z} & 0 \le z < \frac{1}{2} \\ -1 \; \hat{z} & \frac{1}{2} < z < 3 \end{cases} \left[\frac{V}{m}\right]$

c) $\varepsilon_1 = ?$ $\quad$ solve $\vec{D} = \varepsilon \vec{E}$

$\varepsilon_2 = ?$

$\varepsilon_1 = \frac{\vec{E}_1}{\vec{D}_1} \quad \varepsilon_2 = \frac{\vec{E}_2}{\vec{D}_2}$

$\vec{D}_1 = \vec{D}_2$ (independent of material)

since $\rho_s @ z = 1/2 = 0$ since

$\star\star\star \Rightarrow \Rightarrow \Rightarrow$ materials are perfect dielectrics

since $\vec{D} = D_z \hat{z}$ (no tangential)

———— $2\varepsilon_0 = \rho_s$

$\downarrow\downarrow\downarrow$

———— $-2\varepsilon_0 = \rho_s$

$\Downarrow$

$\hat{n} \cdot (\cancel{D_1} - D_2) = 2\varepsilon_0$

$\qquad\;\; 0$

$\varepsilon_1 = \frac{\vec{D}}{-2/3\,\hat{z}}$ $\qquad \vec{D} = -2\varepsilon_0 \hat{z}$

$\varepsilon_1 = \frac{-2\varepsilon_0 \hat{z}}{-2/3\,\hat{z}} = 3\varepsilon_0$ ✓

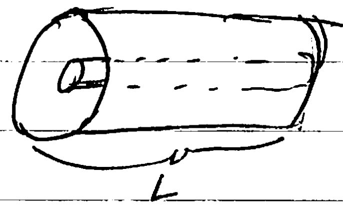
$\varepsilon_2 = \frac{\vec{D}}{-1\hat{z}}$

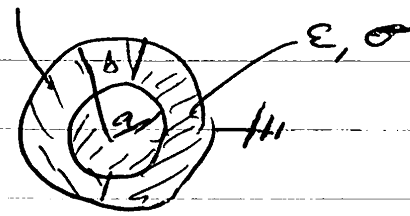
$\varepsilon_2 = \frac{-2\varepsilon_0 \hat{z}}{-1\hat{z}} = 2\varepsilon_0$ ✓

Steady state = time dependent

Fall 15 #6

coaxial cable





$r < a \Rightarrow$ perf. conductor

$\Rightarrow \vec{E} = 0$

$V(r \leq a) = V_0$

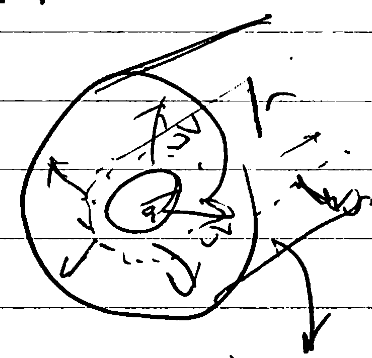
$V(b) = 0$

given $\vec{E}(r) = E_r(r)\,\hat{r}$. $E_r(r) = \frac{V_0}{\ln(b/a)} \frac{1}{r}$

(a) $\vec{J} = ?$

$\vec{J} = \sigma \vec{E}$ A/m^2

(b) total current $I = \int \vec{J} \cdot d\vec{S}$



$= \int_{left} J_{left}\, \hat{r} \cdot \hat{r}\, dS$

$+ \int_{right} J_{right}\, \hat{r} \cdot \hat{r}\, dS \rightarrow d\vec{S} = \hat{r}(r\, d\phi\, dz)$

$= \frac{\sigma V_0}{\ln(b/a)} \int_0^L dz \int_{-\pi/2}^{\pi/2} \frac{1}{r}\, \hat{r} \cdot (\hat{r}\, r\, d\phi)$

$\vec{F} = Q\vec{E}$

Review Notes — Exam 1

$B\left[\frac{Wb}{m^2}\right]$

$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$

Coulomb's law $\vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$

$V = \frac{Q}{4\pi\epsilon_0 r}$

$\vec{D} = \epsilon_0 \vec{E}$ electrical field

displacement field

$\oint_S \vec{D} \cdot d\vec{S} = Q_V = \int_V \rho\, dV$

$\oint_C \vec{E} \cdot d\vec{l} = \int (\vec{\nabla} \times \vec{E}) \cdot d\vec{S}$

stokes thm

$\oint_C \vec{E} \cdot d\vec{l} = 0 \Leftrightarrow \vec{\nabla} \times \vec{E} = 0$

$\nabla \times E = 0$, $\nabla \cdot D = \rho$, $D = \epsilon_0 E$

charge from a line of charge

$\vec{E} = \hat{r}\frac{\lambda}{2\pi\epsilon_0 r}$

$\vec{E} = \hat{x}\frac{\rho_s}{2\epsilon_0}\,\text{sgn}(x)$ const surface density.

$\vec{E} = \hat{x}\frac{\rho x}{\epsilon_0}$ of const volume density ρ infinite slab.

Differential form of Gauss' law:

$\vec{\nabla} \cdot \vec{D} = \rho$ divergence

$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho$

Helmholtz thm

$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

curl $\vec{\nabla} \times \vec{E} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times (E_x, E_y, E_z) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$

$= \hat{x}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) - \hat{y}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) + \hat{z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$

$\vec{\nabla} \times \vec{E} = 0 \Leftrightarrow$ curl-free

gradiant $\nabla V = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right) \Rightarrow \vec{E} = -\vec{\nabla} V$

$\mu_0 = \cancel{(36\pi\times10^{9})^{-1}\ F/m}$ $4\pi\times10^{-7}\ H/m$

$c = 3\times10^{8}\ m/s$ $E = -\frac{dV}{dr}$

$\epsilon_0 \approx \frac{1}{36\pi\times10^{9}}\frac{F}{m}$ charge inside a conductor = 0.

$$\underbrace{\nabla^2 V}_{\text{Laplacian}} = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$\rho = 0 \Rightarrow \nabla^2 V = 0$ Laplace's eqn

Poisson's eqn: $\nabla^2 V = -\frac{\rho}{\epsilon_0}$ if ϵ_r independent of position

$\vec{J} = \sigma\vec{E}$ Ohm's law

polarization $\underset{\text{large}}{\vec{P}} = N_d e d\hat{z} = N_d \underset{\text{smol}}{\vec{p}}$

$\vec{D} = \epsilon\vec{E} = \epsilon_0\vec{E} + \vec{P}$

$C = \frac{2\pi}{\ln\frac{b}{a}}\epsilon$ coaxial cable ~~conductance~~ } per unit length

$G = \frac{2\pi}{\ln\frac{b}{a}}\sigma$ conductance

$\vec{J} = Nq\vec{v}$ current density

$\vec{v} = \frac{q\tau}{m}\vec{E}$ mobility

suceptibility χ_e

Lorentz force: $F = q(\vec{E} + \vec{v}\times\vec{B})$

q [$C = sA$]

E [$N/C = V/m$]

B [$V\cdot s/m^2 = Wb/m^2 = T$]

ρ [C/m^3]

J [A/m^2]

ECE 329 Monday 2 october Lecture 14

Section X
Sub: Waldrop

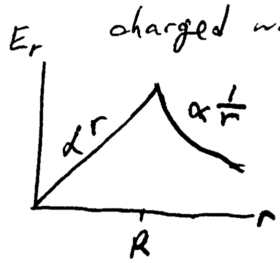
Amperes Law $\oint \vec{H}\cdot d\vec{l} = \int \vec{J}\cdot d\vec{S} = I_{encl}$ $\nabla\times\vec{H} = \vec{J}$
* static fields* $(\vec{H} = \vec{B}/\mu \ [A/m])$

Faraday's* Law $\oint \vec{E}\cdot d\vec{l} = 0$ $\nabla\times\vec{E} = 0$
* static fields*

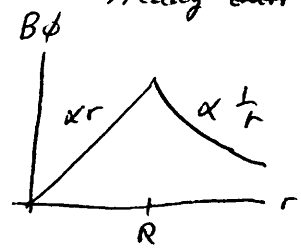
Gauss' Law $\oint \vec{D}\cdot d\vec{S} = \int \rho\, dV = Q_{encl}$ $\nabla\cdot\vec{D} = \rho$
always true $(\vec{D} = \epsilon\vec{E} \ [C/m^2])$

Gauss' Law for $\vec{B}$ $\oint \vec{B}\cdot d\vec{S} = 0$ $\nabla\cdot\vec{B} = 0$
always true

Ex 1 charged wire ρ_0 steady ~~curr~~ current J_0



$\frac{\rho}{\epsilon} \to \mu J$



Recall $\nabla\times\nabla\phi = \phi$
$\nabla\times E = \phi \Rightarrow \vec{E} = -\nabla V \Rightarrow \nabla^2 V = -\rho/\epsilon$

for $\vec{B}$: $\nabla\cdot\nabla\times\vec{A} = \phi$
$\nabla\cdot\vec{B} = \phi \Rightarrow \vec{B} = \nabla\times\vec{A}$

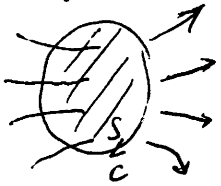
impulse response/
point spread function

$\nabla\times\vec{B} = \underbrace{\nabla\times\nabla\times\vec{A}} = \mu\vec{J}$

$= \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}$ $\Rightarrow \nabla^2\vec{A} = -\mu\vec{J}$

typically $= 0$ (Coulomb "gauge")

Faraday's law: 1840 (one of Maxwell's eqns)



magnetic flux $\Psi = \int_S \vec{B}\cdot d\vec{S}$ [Wb]

$d\hat{S}$ and C follow RH rule

$-\frac{d\Psi}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{S} = \mathcal{E}_{induced} = \oint \vec{E}\cdot d\vec{l}$

- $\vec{E}$ is NOT electrostatic, not conservative, $\oint \vec{E}\cdot d\vec{l} \neq 0$
- $d\hat{S}$ and $d\vec{l}$ follow RH rule

• Lenz's law: induced current generates $\vec{B}$
counteract change in Φ

• $\mathcal{E}$ can be induced around a contour C:

① place C in $\vec{B}(t)$

motional emf { ② moving C in a spatilly varying $\vec{B}$ (rotational emf as well AC current)
③ change shape of C

when derive $\mathcal{E} = \frac{-d\Phi}{dt}$ (assume $d\hat{S}$)
$\Rightarrow$ via RH rule $d\hat{l}$

$\mathcal{E} > 0$ then induced current in same direction as $d\vec{l}$

$\mathcal{E} < 0$ " " opposite direction

---

in general: $\mathcal{E} = \frac{W}{q} = \oint \frac{\vec{F}}{q} \cdot d\vec{l} = \underbrace{\oint \vec{E} \cdot d\vec{l}}_{\text{moving frame}} + \underbrace{\oint (\vec{v} \times \vec{B}) \cdot d\vec{l}}_{\text{in lab frame}}$

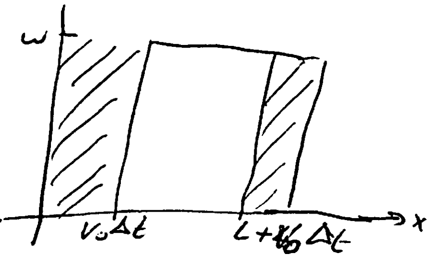
$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Faraday: $\mathcal{E} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{S}$

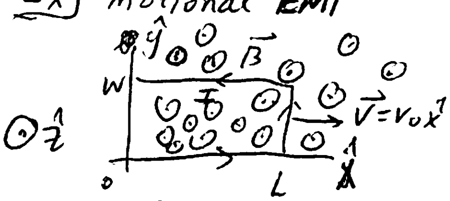
so: $\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{S} - \oint_C (\vec{v} \times \vec{B}) \cdot d\vec{l}$

via stoke's thm $\int_S (\nabla \times \vec{E}) \cdot d\vec{S} = -\int_S \frac{\partial B}{\partial t} \cdot d\vec{S}$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$ Faraday's law



Ex] motional EMF

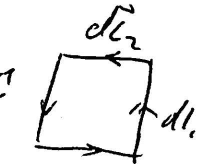


CCW L
(Lenz' law)

$\vec{B} = B_z(x)\hat{z}$

Solution #1: $\frac{d\Phi}{dt} \simeq \frac{\Phi(t+\Delta t) - \Phi(t)}{\Delta t}$

Solution #2: $\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$ $\quad v = v_0 \hat{x}$, $\vec{B} = B\hat{z}$



Sub: Waldrop | Wednesday 4 October Lecture 14 pt 2/15

$\Psi(t+\Delta t)$ = original + new piece − old piece

Lenz's law

rail gun

$\mathcal{E} = \oint \frac{\vec{F}}{q} \cdot d\vec{l}$ emf is a force per unit charge across a loop

3 different ways to find an induced current

$\mathcal{E}$ emf [Wb/s]

Lec 15

Recall: currents $I \rightarrow \vec{B} \rightarrow \Psi$

$$-\frac{d\Psi}{dt} = \mathcal{E} = IR$$

mutual inductance

self inductance

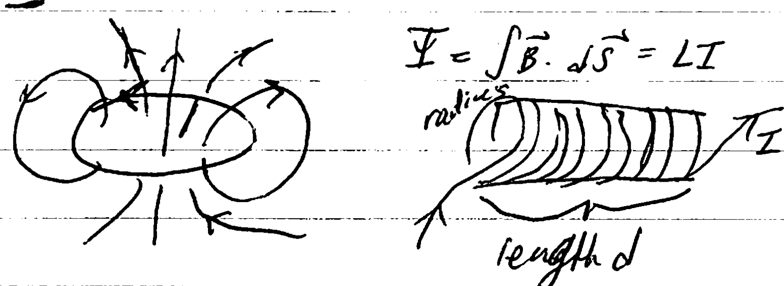
INDUCTANCE $L = \frac{\Psi}{I}$ "self inductance of path c"

$$\mathcal{E} = \frac{-d\Psi}{dt} = -L\frac{dI}{dt}$$

$v(t) = L\frac{dI}{dt}$

EX] SOLENOID.

$\Psi = \int \vec{B} \cdot d\vec{S} = LI$



$\vec{B}$ for a solenoid? $\vec{B} = (B_r, B_\phi, B_z)$

solve for B_ϕ: $\oint \vec{H} \cdot d\vec{l} = \oint H_\phi \hat{\phi} \cdot \hat{\phi}\, dl = H_\phi 2\pi r = I_{encl}$

$= \int \vec{J} \cdot d\vec{S} = \int \vec{J} \cdot \hat{z}\, ds$

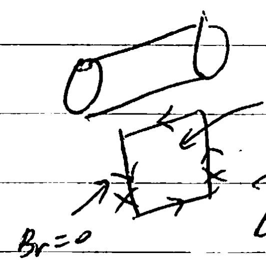
ϕ $= 0$

$\therefore H_\phi = B_\phi = 0$

solve for B_r: $\oint \vec{B}\cdot d\vec{S}=0 = \int B_r \hat{r}\cdot\hat{n}\, dS=0$

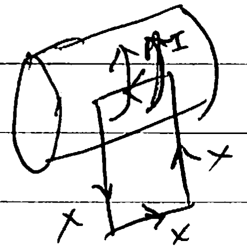
$\therefore\ B_r=0$

solve for B_z: $\oint \vec{H}\cdot d\vec{l} = I_{encl}$



$I_{encl}=0$

two pieces must be equal and opposite. Since $B_z(r>a)=0$ at ∞, the field is 0, then all B outside solenoid is 0.

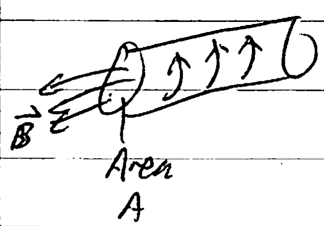


$\oint \vec{H}\cdot d\vec{l} = I_{encl}$

$H_z = IN = I\, {}^{n}/_{d}$

density of loops

length of solenoid



$\Phi = \mu INA$

$L = \frac{\Phi}{I} n = N^2 \mu A d\ [H]$

total of loops

absorbed power $P = VI = \left(L\frac{dI}{dt}\right)I = \frac{d}{dt}\underbrace{\left(\tfrac{1}{2}LI^2\right)}_{[J]\text{ stored energy}} = [W] = [J/s]$

For solenoid: $\frac{1}{2}LI^2 = \frac{1}{2}N^2\mu A d I^2$

density of loops

length of solenoid

$\left(\vec{H} = \frac{\vec{B}}{\mu}\right)$

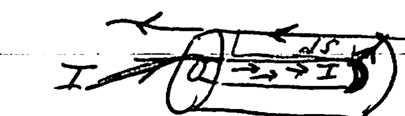
$= \frac{|B_z|^2}{2\mu}Ad = \frac{1}{2}\mu|H_z|^2 Ad = \frac{1}{2}(\mu\vec{H}\cdot\vec{H})Ad$

$= \underbrace{\tfrac{1}{2}\vec{B}\cdot\vec{H}}_{J/m^3}(Ad)$

stored energy density

in a capacitor: stored energy density $= \frac{1}{2}\vec{D}\cdot\vec{E}$

Ex] shorted co·ax cable



$L = \frac{\Psi}{I} = \frac{\int \vec{B}\cdot d\vec{s}}{I}$ magnetic field strength azimuthal

$\vec{B} = B_\phi \hat{\phi} = \hat{\phi}\frac{\mu I}{2\pi r}$ [Wb/m^2] inner radius = a, outer radius = b

$$\Psi = \int \vec{B}\cdot d\vec{s} = \int_a^b dr \int_0^\ell dz \frac{\mu I}{2\pi r} = \frac{\mu I}{2\pi}\ell \underbrace{\int_a^b \frac{1}{r}\,dr}_{\ln(b/a)}$$

$$L_{co\cdot ax} = \mu \ell \frac{\ln(b/a)}{2\pi}$$

$$\varepsilon_{co\cdot ax} = \varepsilon \ell \frac{2\pi}{\ln(b/a)}$$ define GF = "geometric factor"

define $\mathcal{L} = \frac{L}{\ell}$ $\quad \mathcal{C} = \frac{C}{\ell}$ per unit length

$\quad = \mu\left(\frac{1}{GF}\right) \quad = \varepsilon(GF)$

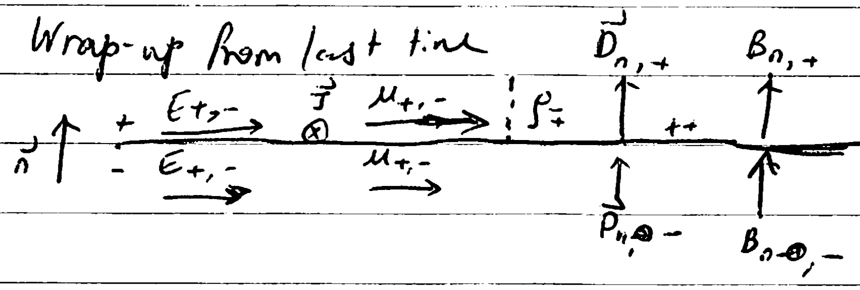
$\mu\varepsilon = 1/c^2$ $\qquad \mathcal{G} = \sigma(GF)$

Lee
Section X

Mon 9 Oct Lecture 17

Magnetization current, magnetic fields in material media

Wrap-up from last time

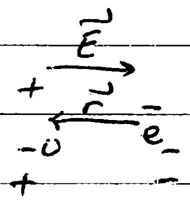


Recall from lectures 8 and 11

$\vec{D} = \varepsilon \vec{E} = \varepsilon_0 \vec{E} + \vec{P} \longrightarrow$ polarization field

$\vec{P} = \varepsilon_0 \chi_e \vec{E}, \quad \varepsilon = \varepsilon_0(1+\chi_e) = \varepsilon_r \varepsilon_0$

$\underbrace{\varepsilon_r}_{\text{relative permittivity}} = 1 + \underbrace{\chi_e}_{\text{electric susceptibility}}$



$\vec{p} = -q\vec{r}$
dipole moment

AC current, $\vec{J}_p = \frac{d\vec{P}}{dt}$ "displacement current"
bound charges

Objective: analogous fields/currents in materials from external magnetic fields

$\vec{M}$, magnetization

$\nabla\times\vec{M}$ magnetization current density (bound charges)

χ_m magnetic susceptibility

$\mu = \mu_0(1+\chi_m)$ permeability

Separate out force from bound charge

$\underbrace{\rho = \rho_f}_{\text{force}} \underbrace{- \nabla\cdot\vec{P}}_{\text{bond}} \qquad \vec{J} = \vec{J}_f + \frac{\partial \vec{P}}{\partial t} + \nabla\times\vec{M}$

plug into Gauss's, Ampere's Laws microscopic form

in a vacuum

$$\nabla\cdot\varepsilon_0\vec{E} = \rho \Rightarrow \nabla\cdot(\varepsilon_0\vec{E}+\vec{P}) = \rho_f$$

$$\nabla\times\mu_0^{-1}\vec{B} = \vec{J} + \frac{\partial\varepsilon_0\vec{E}}{\partial t} \Rightarrow \nabla\times(\mu_0^{-1}\vec{B}-\vec{M}) = J_f + \frac{\partial}{\partial t}(\varepsilon_0\vec{E}+\vec{P})$$

$$\vec{D} = \varepsilon_0\vec{E}+\vec{P} = \varepsilon\vec{E}$$

$$\vec{H} = \mu_0^{-1}\vec{B} - \vec{M} = \mu^{-1}\vec{B}$$

Maxwell's Eq'ns in a material medium

$$\left.\begin{aligned} \nabla\cdot\vec{D} &= \rho \\ \nabla\cdot\vec{B} &= 0 \\ \nabla\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{H} &= \vec{J} + \frac{\partial B}{\partial t} \end{aligned}\right\} \quad \begin{aligned} \vec{D} &= \varepsilon\vec{E} \\ \vec{B} &= \mu\vec{H} \end{aligned}$$

ρ and J $\Rightarrow$ implicitly due to free charges

Bound charge densities must satisfy continuity

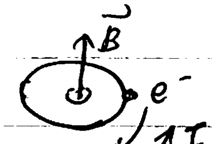
$$\frac{\partial\rho}{\partial t} + \nabla\cdot\vec{J} = 0$$

$$\rho = \rho_b = -\nabla\cdot\vec{P} \quad \leftarrow \text{see lecture 8}$$

$$\vec{J} = \vec{J}_b = \frac{\partial\vec{P}}{\partial t}$$

Bound electrons in material can conduct divergence-free currents in their closed-loop orbitals

Add a curl term to $\vec{J}_b$ while still satisfying continuity, since divergence of curl of any field $= 0$



Experimentally: $\vec{M} = \mu_0^{-1}\vec{B} - \vec{H}$

$$\vec{M} = \chi_m\vec{H}$$

$$\vec{B} = \mu_0(1+\chi_m)\vec{H} = \mu\vec{H}$$

$$\mu = \mu_0(1+\chi_m)$$

tunnel charge - SSD
* Why are cafeteria forks magnetized?

For many materials, $|\chi_m| << 1$

<u>Diamagnetism</u> if $\chi_m < 0$

Applied field induces electron orbital momentum $\vec{B}$ that partly cancels magnetic field inside material. Wood, glass, plastics

<u>Paramagnetic</u> if $\chi_m > 0$

Some spin angular momentum from inner, unfilled shells

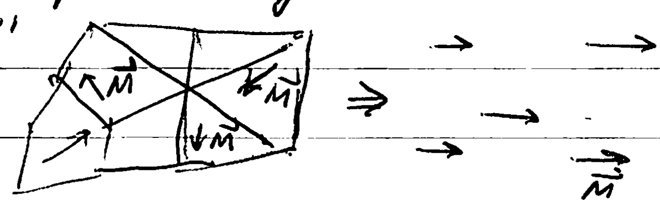
<u>External</u> $\vec{B}/\vec{H}$ field co-aligns spin angular momentum.

Al: $\chi_m = 2.1 \times 10^{-5}$

<u>Ferromagnetic</u> $\chi_m >> 0$ Fe, Co, Ni unpaired 3d electron

$\vec{M}$ arises spontaneously

"Domains"



<u>external</u> magnetic field aligns domains

$\vec{B}$ varies non-linearly w/ $\vec{H}$ and depends on past values

$$\vec{B} = \vec{B}(\vec{H})$$

impedence matching

section X
Lee

Wed 11 Oct Lecture 18

Wave equation

Objective: introduce "plane" waves as a solution to Maxwell's eqns

Waves in free space

Simplest solution to Maxwell's eqn: plane wave

$\vec{P} = \vec{M} = 0$

$\rho = \vec{J} = 0 = \sigma$

ε_0, μ_0

1 eqn – $\nabla \cdot \vec{D} = 0$

1 eqn – $\nabla \cdot \vec{B} = 0$

3 eqn – $\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$ spatial variations in $\vec{E}$ cause time-variations in $\vec{B}$

3 eqn – $\nabla \times \vec{H} = \frac{d\vec{D}}{dt}$

8 eqns

$\vec{E}, \vec{D}, \vec{B}, \vec{H} = 12$ unknowns

14 eqns total

3 eqn → $\vec{D} = \varepsilon_0 \vec{E}$ ⇒

3 eqn → $\vec{H} = \vec{B}/\mu$

6 eqns

Light, radio waves, Wi-Fi, etc. are non-zero fields solutions that obey Maxwell's eqns

EM spectrum

low frequency
long λ

high frequency
low λ

longitudinal vs — waves

"curly" - Lecture 18 p.3

mks units

from Gauss' law in free space

$$\nabla\times(\nabla\times\vec{E}) = \nabla(\overbrace{\nabla\cdot\vec{E}}^{0}) - \nabla^2\vec{E}$$

$$\nabla\times(\nabla\times\vec{E}) = -\nabla^2\vec{E}$$

Curl of Faraday's Law

$$\nabla\times\left[\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}\right] \Rightarrow -\nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}\nabla\times\vec{H}$$

Sub in Ampere's Law: $\nabla\times\vec{H} = \varepsilon\frac{\partial\vec{E}}{\partial t}$

$$\nabla^2\vec{E} = \varepsilon\mu\frac{\partial}{\partial t}\frac{\partial\vec{E}}{\partial t}$$

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

1D scalar wave eqn

$\vec{E}(\vec{r},t) = \hat{x}E_x(z,t)$ → varies in time, varies in z-direction, but independent of x, y

polarized in x-direction

$$\frac{\partial^2 E_x}{\partial z^2} = \mu\varepsilon\frac{\partial^2 E_x}{\partial t^2}$$

$E_x = \cos(\omega(t - \sqrt{\mu\varepsilon}z))$ → propagates in $\hat{z}$ direction

$E_x = \cos(\underbrace{\omega(t + \sqrt{\mu\varepsilon}z)}_{rad/s})$

$\omega \equiv$ oscillation frequency $\left[\frac{rad}{s}\right]$

$\sqrt{\mu\varepsilon} = \frac{s}{m}$

recall: $\frac{1}{\sqrt{\mu\varepsilon}} = \frac{m}{s}$ propagation speed of light 300 m/μs

~~negative~~

propagates $-\hat{z}$

$E_x = \cos(\omega(t \mp \frac{z}{v}))$ for $v = \frac{1}{\sqrt{\mu\varepsilon}}$

$$\nabla\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = \hat{y}\frac{\partial E_x}{\partial z}$$

Transverse electromagnetic (TEM) waves

$$= \pm\hat{y}\sin(\omega(t \mp \frac{z}{v}))\frac{\omega}{v}$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\vec{H} = \pm \hat{y} \sqrt{\frac{\varepsilon}{\mu}} \cos\left(\omega\left(t \mp \frac{z}{v}\right)\right)$$

$+\hat{y} \rightarrow -z/v$

$-\hat{y} \rightarrow +z/v$

$$\vec{E} = \hat{x} f\left(t \mp \frac{z}{v}\right)$$

$$\vec{H} = \pm \hat{y} \frac{f\left(t \mp \frac{z}{v}\right)}{\eta}, \quad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

"intrinsic impedance" in ohms

Maxwell's eqns $\Rightarrow$ LTI $\Rightarrow$ by Fourier, sums of cosines also obey Maxwell's eqns

$\Rightarrow$ any signal will as well

$$f(t) = \sum_n A_n \cos(\omega_n t + \theta_n)$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} \, d\omega$$

$f(t)$ can be <u>arbitrary</u>

$\vec{E}, \vec{H} \propto f\left(t \mp \frac{z}{v}\right)$ d'Alembert wave solutions

$$\vec{E} = \hat{y} f\left(t \mp \frac{z}{v}\right)$$

$$\vec{H} = \mp \hat{x} \frac{f\left(t \mp \frac{z}{v}\right)}{\eta}$$

z-polarized waves propagating in z-direction $\Rightarrow$ violates $\nabla \cdot \vec{E} = 0$

ECE 329 HKN Review Session

<u>Magnetostatics ($\frac{dI}{dt} = 0$)</u>

Lorentz force

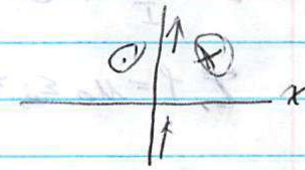
Biot-Savart Law

<u>Ampere's Law</u>

Current Density (J): $\vec{I}_{enc} = \oiint_S \vec{J} \cdot d\vec{S}$

Magnetic Field Intensity (H): $\vec{B} = \mu \vec{H}$

Sheet of charge



Ampere's Law: use RHR

Wire

Sheet of current

Solenoid

<u>Continuity Equation and Maxwell's Equation:</u>

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot \vec{J}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Maxwell's eqns:

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

<u>Non-Conservative Fields</u>

Magnetic Flux: $\psi = \oiint_S \vec{B} \cdot d\vec{S}$

Electromotive force (emf): $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \neq 0$

<u>Non-Zero Flux</u>

1. Area or $B \cdot dS$ is changing (area grows, wire rotates)
2. Time-Varying B
3. Position changing B and $v \neq 0$

$$I = \frac{\mathcal{E}_{mf}}{R} = -\frac{1}{R}\frac{\partial \psi}{\partial t}$$

Current <u>opposes</u> increases in magnetic field

<u>Inductance (L)</u>

The tendency of a device to resist changes in current [H]

$$L = \frac{\psi}{I}, \quad C = \frac{Q}{V}$$

$$\mathcal{L}\,\ell = \mu_0 \varepsilon_0 = \frac{1}{c^2}$$

<u>Boundary Conditions</u>

Important! Put on note sheet.

<u>Materials</u>

Diamagnetic, Paramagnetic, Ferromagnetic

$\vec{B}_{tot} = \vec{\mu}_0 (\vec{H}_{ext} + \vec{M})$

$\vec{M} = \chi_m \vec{H}_{ext}$

<u>Wave Equations</u>

Helmholtz eqns

D'Alembert solutions

Useful eqns

<u>Poynting's Theorem</u>

Average Poynting Vector

$$\langle \tilde{S} \rangle = \frac{1}{2} Re\{ \tilde{\vec{E}} \times \tilde{H}^* \} = \frac{|\vec{E}|^2}{2\eta} = \frac{|\tilde{H}|^2 \eta}{2} \left[\frac{W}{m^2}\right]$$

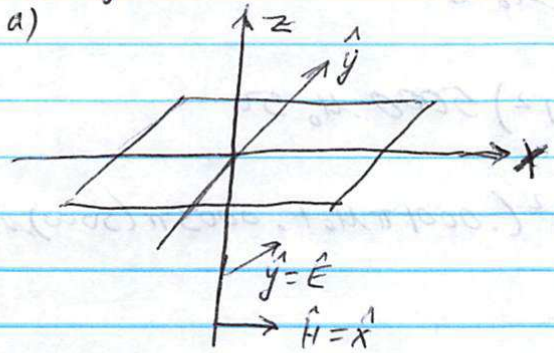
<u>Plane Wave Sources</u>

1. Direction of H given by RHR, $|\vec{H}| = \frac{|\vec{J}_s|}{2}$
2. E points opposite of J_s
3. Wave propagates away from source
4. $|E| = \eta |H|$, E and H always perpendicular
5. Solve for Poynting's Vector $\vec{S} = \vec{E} \times \vec{H}$

ECE 329 HKN Review Session

Spring 2016 #1

a)



b) $4\cos(\omega t \mp \beta z)$

$$V_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^{14}}{\pi \times 10^6} = 2\times10^8 \text{ m/s}$$

c) $\mu = \mu_0$

V_p known

$$V_p = \frac{1}{\sqrt{\mu \varepsilon}}$$

$$V_p^2 \mu_0 \varepsilon_0 = 1/\varepsilon_r \Rightarrow \varepsilon_r = \frac{1}{V_p^2 \mu_0 \varepsilon_0} = \frac{9}{4}$$

d) IV, by elimination

g) Magnetic fields change signs on opposite sides of the plane

h) $\hat{n} \times (H^+ - H^-) = J_s$

Spring 2016 #2

$H = IN\hat{z}$

$H = 1 \cdot 50 \cdot \hat{z} = 50 \frac{A}{m}\hat{z}$

$B = \mu H = \mu_r \mu_0 H = 5000\,\mu_0 (50)$

$= 250000\,\mu_0 \frac{Wb}{m^2}\hat{z}$

b)

$L = N\frac{\psi}{I}$, $\psi = \oint\oint B \cdot dS = BA$

$A = (.02)^2\pi$

$$L = \frac{50 \times .02^2 \pi \times 2.5\times10^5 \mu_0}{I=1}$$

c) $\Psi_{\text{free space}} = BA = \pi(.01)^2 \mu_0\, 50$

$\Psi_{\substack{\text{other}\\ (\text{iron})}} = BA = \pi(.02^2 - .01^2)\, 5000\, \mu_0\, 50$

$\dfrac{N f(\Psi_{tot})}{I=1}$ $\quad L = 50^2(.0001\pi\mu_0 + .0003\pi(5000)\mu_0)$

<u>Exam 2 #4</u>

a) $J_s = f(t)\,\hat{y}$

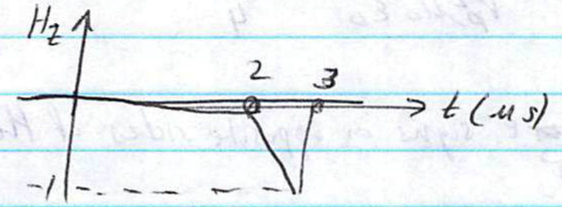
$\vec{E} = -\hat{J}_s \qquad E = \eta_0 H$

$|H| = \dfrac{|J_s|}{2}, \ |E| = \eta_0 \dfrac{J_s}{2} \qquad |E \times H| = \hat{S}$

$H(r_0, t) = -\hat{z}\ \frac{1}{2} f(t - 1\mu s)$

$E(r_0, t) = -\frac{1}{2}\hat{y}\, f\left(t - \frac{X_0}{c}\right)\eta_0 = -\frac{\eta_0}{2} f(t - 1\mu s)\,\hat{y}$

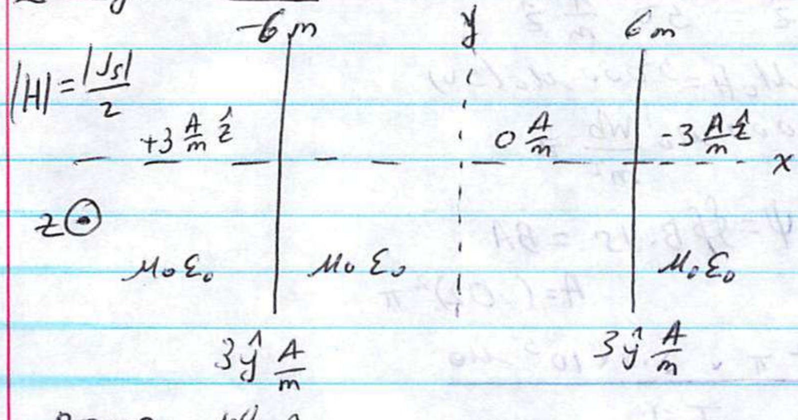
b) $H = -\hat{z}\ \frac{1}{2} f(t - 1\mu s)$



linear transformation
flip, scale, shift

c) Just consult graph for b)

<u>Spring 2016 #4</u>



$B = 3\mu_0 \frac{Wb}{m^2}\hat{z} \qquad B = 0\ \frac{W}{m^2} \qquad B = -3\mu_0 \frac{Wb}{m^2}\hat{z}$

b) $\mathcal{E}_{mf} = -\dfrac{\partial \Psi}{\partial t} \quad \Psi = BA \quad \mathcal{E}_{mf} = 0$
$I = 0$

c) $\Psi = \iint B \cdot dS \quad \iint 3\, dx\, dy$

$\Psi = 3\cos(2\pi t)\mu_0$

$B = \mu_0 H$

$f = 1$
$\omega = 2\pi$

$\mathcal{E}_{mf} = +\mu_0\, 3(2\pi)\sin(2\pi t)\, V$

$I = \mu_0\, 3\pi \sin(2\pi t)\, A$

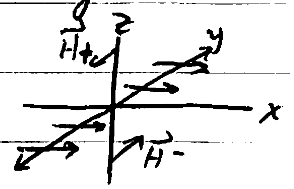
Lee
Section X

Mon 16 Oct Lecture 20

Objective: Wave propagation from current sheets and Poynting thm

Generation of plane TEM waves by time-varying current sheets

$\vec{J_s} = \hat{x} J_x$ A/m flowing on $z=0$ surface



$\vec{J_s} = \hat{x} J_x$

$\vec{H}(z) = \mp \hat{y} \frac{J_x}{2} \frac{A}{m}$ for $z \gtrless 0$ (Lecture 13)

Suppose $J_x = J_x(t)$

$\vec{H}(z,t) \approx \mp \hat{y} \frac{J_x(t)}{2}$ $z \gtrless 0$ very close to origin

$\vec{H}(z,t) = \mp \frac{J_x(t \mp \frac{z}{v})}{2}$ for $z \gtrless 0$, includes time shift

Next use d'Alembert solution

$$\vec{E}(z,t) = -\hat{x} \frac{\eta}{2} J_x(t \mp \tfrac{z}{v})$$

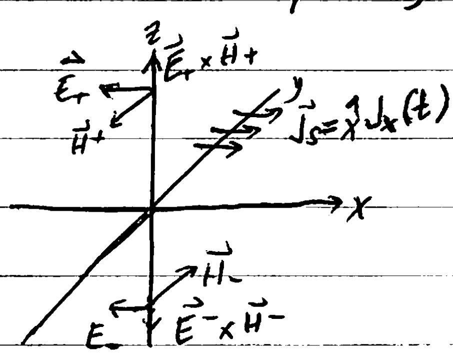
$\vec{S} = \vec{E} \times \vec{H}$

$+\hat{z} = -\hat{x} \times -\hat{y}$

$z \gtrless 0$

$\vec{H}$ has odd symmetry about $z=0$

$\vec{E}$ has even symmetry and is anti-parallel to J_s



$z=0$ as a boundary

Note: $\vec{E}^{\pm}$ is tangential and shouldn't change = continuous

$\vec{H}^{\pm}$ is discontinuous across ss

. . .

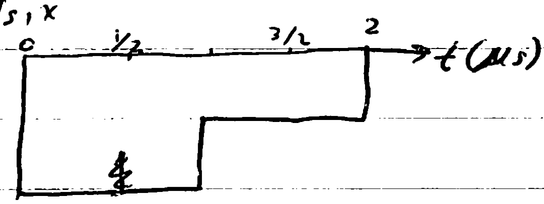
Current sheet at $z=0$, $\vec{J_s}(t)=J_s(t)(-\hat{x})$

Given $v=c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}=300\,\text{m}/\mu s$

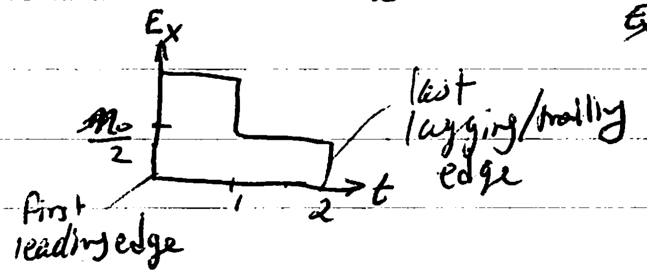
$J_s(t)=2\,\text{rect}\left(\frac{t-1/2}{1}\right)+1\cdot\text{rect}\left(\frac{t-3/2}{1}\right)$

$\eta=\eta_0$

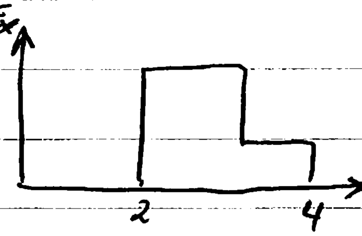
(a) $J_{s,x}$



(b) $\vec{E}(0,t)=\hat{x}\,\frac{\eta_0}{2}\,J_s(t)$

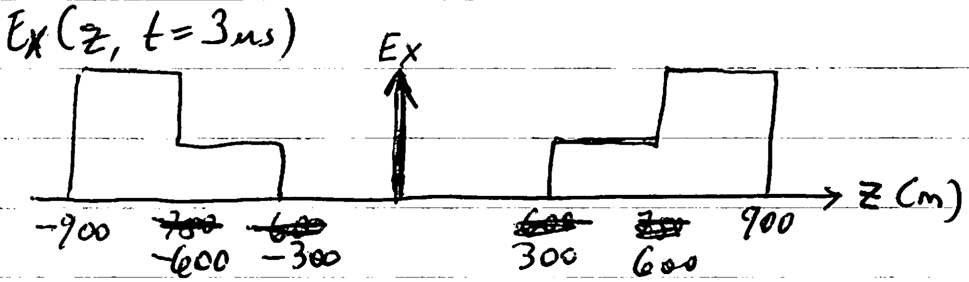


(c) $E_x(600\text{m},t)$



Same as at $z=0$, but w/ propagation delay

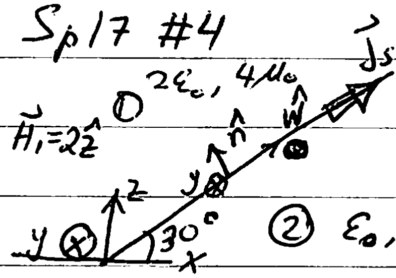
(d) $E_x(z,t=3\mu s)$



Wed 18 Oct Lecture — Waldrop

Midterm Review Session

Sp17 #4



$\hat{n} = -\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{z}$

$\hat{w} = \frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{z}$

$\vec{J}_s = 3\hat{w}$

$\hat{n}\cdot(\mu_1\vec{H}_1 - \mu_2\vec{H}_2) = 0$

$\hat{n}\cdot(\vec{B}_1 - \vec{B}_2) = 0$

$\hat{n}\times(\vec{H}_1 - \vec{H}_2) = \vec{J}_s$

$\hat{n}$ component: $(\mu_1 H_{1n} - \mu_2 H_{2n} = 0)$ (D)

y-component $[\hat{n}\times(\vec{H}_1 - \vec{H}_2)]_y = H_{1y} - H_{2y} = 0$ (A)

w-component $[H_{1w} - H_{2w}] = J_{sw}$

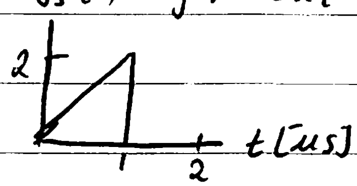
Sp15 #5

$J_s(t) = \hat{y}\, A\, t\, [u(t) - u(t-\tau)]$ ~~$E(z,t) = \ldots$~~

τ 1 μs

$\vec{J}_s$ @ z=0

μ_0, ε_0



c = 300 m/μs

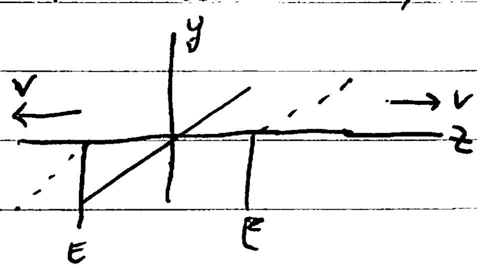
$\eta_0 = 120\pi$

$t \to (t \mp z/c)$

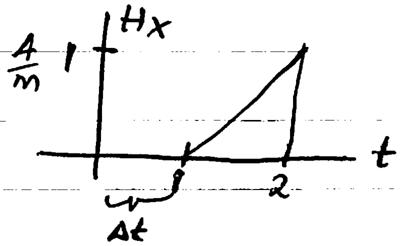
$$\vec{E}(z,t) = -\hat{y}\left(\frac{\eta_0 A}{2}\right)(t \mp \tfrac{z}{c})[u(t \mp z/c) - u(t \mp z/c - \tau)]$$

120π

$\vec{H}(z,t) = \pm\hat{x}\frac{A}{2}(\cdots)$, $z \gtrless 0$



(b) plot $\vec{H}(z,t)$ @ $z = 300$ m



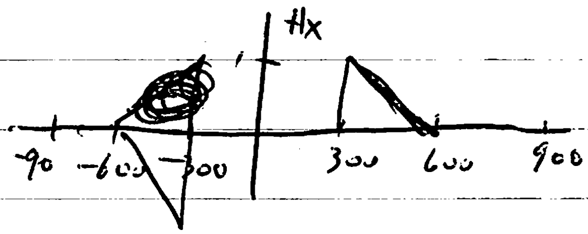
$\Delta t = \frac{300\text{ m}}{c} = 1\ \mu s$

(c) 5 m² area — instantaneous power

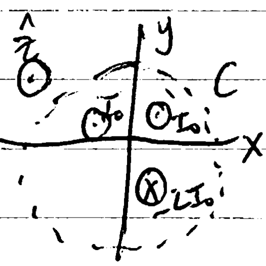
$\vec{S} = \vec{E} \times \vec{H}$ — $\vec{S}(A)$

(d) plot $\vec{H}(z,t)$ @ $t = 2\ \mu s$

$\Delta z = 300\text{ m}/\mu s \times 2\ \mu s$
$= 600$ m



Sp17 1a

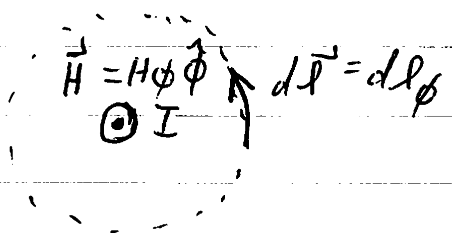


T/F: $\oint_C \vec{H} \cdot d\vec{l} = 0 = I_{encl} \overset{\checkmark}{=} 0$ T

T/F: $|\vec{H}| = 0$ along C F

T/F: $H_z = 0$ along C T

$\oint \vec{H} \cdot d\vec{l} = I$ $\oint H_\phi \, dl$



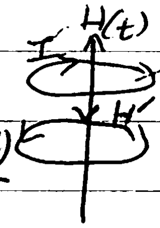
$H_\phi = I/2\pi r$

Lenz's law

Faraday's law

Sp 15 1a:

T/F: induced current from $\vec{H}(t)$ produces $\vec{H}'$ in opposite dir of $\vec{H}$



$0 < \frac{dI(t)}{dt}$

$\Phi(t) = \int \vec{B} \cdot d\vec{S}$

$\mathcal{E} = -\frac{d\Phi}{dt} < 0$

Lee

Fri 20 Oct Lecture 21 Section X

Objective: Examples using Poynting theorem, phasor representation of monochromatic waves

$\vec{S} = \vec{E} \times \vec{H}$

Poynting thm: $\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon \vec{E}\cdot\vec{E} + \frac{1}{2}\mu \vec{H}\cdot\vec{H}\right) + \vec{\nabla}\cdot(\vec{E}\times\vec{H}) + \vec{J}\cdot\vec{E} = 0$

Conservation Law

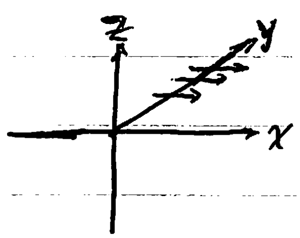
Energy stored in the form of $\vec{E}$ and $\vec{H}$ fields

energy transported (e.g. by waves) per volume

Joule heating, absorbed energy per volume

Monochromatic waves: $z = 0$ $\vec{J}_s = \hat{x} f(t) = \hat{x}\, 2\cos(\omega t) \frac{A}{m}$

a) $\vec{E}(z,t)$, $\vec{H}(z,t)$ for $z \gtrless 0$? b) $\vec{S}$? c) $\vec{J}_s \cdot \vec{E}$ at $z=0$?



$\vec{J}_s = \hat{x} f(t)$ What is $\vec{E}$ polarized in? $-\hat{x}$

$\vec{E} = -\eta\cos(\omega t \mp \beta z)\,\hat{x}$

(b) $\vec{S} = \vec{E}\times\vec{H} = \pm\eta\cos^2(\omega t \mp \beta z)\,\hat{z}$

(c) At $z=0$, $\vec{E}(0,t) = -\eta\cos(\omega t)\,\hat{x}$

$\vec{J}_s \cdot \vec{E} = -2\eta\cos^2(\omega t)$

Time-averaged Poynting vector

$\langle\cos^2(\omega t + \phi)\rangle = \langle\frac{1}{2}[1 + \cos(2\omega t + 2\phi)]\rangle$

$= \frac{1}{2}$

$\langle\vec{E}\times\vec{H}\rangle = \pm\eta\frac{1}{2}\hat{z} = \pm 60\pi\,\hat{z}$

Time average $\vec{J}_s \cdot \vec{E} = 120\pi\ \frac{W}{m^2}$

Poynting vector $\vec{S}$ [=] W/m² J_s [=] A/m J [=] A/m²

Example $\vec{H} = 3\cos(\omega t + \beta y - \frac{\pi}{3})\hat{x} - 3\sin(\omega t + \beta y + \frac{\pi}{6})\ \frac{A}{m}$

in vaccuum, $v = c = 1/\sqrt{\mu_0 \varepsilon_0}$, $\eta = \eta_0$; $\omega = 2\pi \cdot 10^8\ \frac{rad}{s}$

a) Direction of propagation $-\hat{y}$

b) $\beta = ?$ $\quad \frac{\omega}{\beta} = c$ ← "dispersion relation"

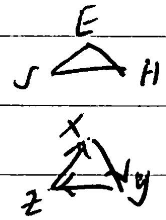
$$= \frac{2\pi \cdot 10^5\ rad/s}{3\times 10^8\ m/s} = \frac{2\pi}{3}\ \frac{rad}{m}$$

c) $\lambda = ? = \frac{2\pi}{\beta} = 3m$ "radio wave"

d) $\vec{E} = ?$ $\quad \frac{|\vec{E}|}{|\vec{H}|} = \eta$

$$\vec{E} = 3\eta\cos(\cdots)(-\hat{z}) - 3\eta\sin(\omega t + \cdots)\ \hat{x}$$

$\_\_ \times \hat{x} = -\hat{y}$ $\qquad \hat{x} \times \hat{z} = -\hat{y}$



e) Instantaneous power crossing $1\,m^2$ area of xz plane?

$$\vec{S} = (-\hat{y})[9\eta_0\cos^2(\omega t + \beta y - \tfrac{\pi}{3}) + 9\eta_0 \underbrace{\sin^2(\omega t + \beta y + \tfrac{\pi}{6})}_{\cos^2(\omega t + \beta y + \frac{\pi}{6} - \frac{\pi}{2})}]$$

$$= (-\hat{y})[18\eta_0\cos^2(\omega t + \beta y - \tfrac{\pi}{3})]$$

$$\int \vec{S}\cdot d\vec{S} = \int \vec{S}\cdot\hat{y}\, dS = -18\eta_0\cos^2(\omega t + \beta y - \tfrac{\pi}{3})\Big|_{1m^2}$$

$\vdots$

$$= -9\eta_0$$

Damped oscillations

Mon 23 Oct Lecture 22

Phasor representations of monochromatic waves

~~Phasor~~ Phasor review

① $v(t) = V_0 \cos(\omega t + \phi)$

$\tilde{v} = V_0 e^{j\phi}$

② $\tilde{v} = V_0 e^{j\phi}$

$v(t) = Re[\tilde{v} e^{j\omega t}]$

③ a) Amplitude of $\tilde{v} = |\tilde{v}|$

$\tilde{v} = x + jy \quad |\tilde{v}| = \sqrt{x^2+y^2}$

b) Phase angle

$\theta = \tan^{-1}(y/x)$

$e^{j\theta} = \cos\theta + j\sin\theta$ Euler's identity

$\tilde{z} = x + jy \qquad \theta = \tan^{-1}(y/x)$

$= M(\cos + j\sin\theta) \qquad M = \sqrt{x^2+y^2}$

$= Me^{j\theta}$

$\tilde{z} = \dfrac{A+jB}{C+jD} \Rightarrow \tilde{z}^* = \dfrac{A-jB}{C-jD}, \quad \tilde{z} = Me^{j\theta} \Rightarrow \tilde{z}^* = Me^{-j\theta}$

$\vec{E} = E_0 \cos(\omega t \mp \beta z)\,\hat{x}\,\frac{V}{m} \qquad \tilde{E} = E_0 e^{\mp j\beta z}\,\hat{x}\,\frac{V}{m}$

Given: Field — Phasor

$\vec{E} = \cos(\omega t + \beta y)\hat{z} \qquad \tilde{E} = e^{j\beta y}\hat{z}$

$\tilde{H} = -\frac{e^{j\beta y}}{\eta}\hat{x}$

$\vec{H} = \sin(\omega t - \beta z)\hat{y} \qquad \tilde{H} = -je^{-j\beta z}\hat{y}$

$\vec{H}(t) = Re(-je^{-j\beta z} e^{j\omega t})\hat{y}$

$= Re(-je^{j(\omega t - \beta z)})\hat{y}$

$= Re(-j(\cos + j\sin))\hat{y}$

$\tilde{E} = -j\eta e^{-j\beta z}\,\hat{x}$

$= \sin(\omega t - \beta z)\hat{y}$

$\tilde{S} = \tilde{E} \times \tilde{H}^*$

Complex Poynting vector

→ points in propagation direction

Time-average of Poynting vector

Recall: $\langle p(t) \rangle = \langle v(t)\, i(t) \rangle = \frac{1}{2} Re\{\tilde{V}\, \tilde{i}^*\}$

$$\langle \vec{E} \times \vec{H} \rangle = \frac{1}{2} Re\,[\tilde{E} \times \tilde{H}^*], \quad \langle \vec{J_s} \cdot \vec{E} \rangle = \frac{1}{2} Re[\tilde{J_s} \cdot \tilde{E}^*]$$

(see "proof" in course notes)

Building up to damped waves in conducting media

$$\vec{\nabla} \cdot \tilde{D} = \tilde{\rho} \qquad \vec{\nabla} \times \tilde{E} = -j\omega \tilde{B} \quad \left(\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}\right)$$

$$\vec{\nabla} \cdot \tilde{B} = 0 \qquad \vec{\nabla} \times \tilde{H} = \tilde{J} + j\omega \tilde{D}$$

$$\frac{\partial}{\partial t} \tilde{V} = j\omega \tilde{V}$$

$$\tilde{D} = \epsilon \tilde{E} \qquad \tilde{J} = \sigma \tilde{E}$$

$$\tilde{B} = \mu \tilde{H}$$

ϵ, μ, σ could be functions of ω

Let $\tilde{\rho} = \tilde{J} = 0$ $\quad \vec{\nabla} \cdot \tilde{E} = 0 \quad \vec{\nabla} \times \tilde{E} = -j\omega\mu \tilde{H}$

$$\vec{\nabla} \cdot \tilde{H} = 0 \qquad \vec{\nabla} \times \tilde{H} = j\omega \epsilon \tilde{E}$$

$\vdots$

$$\vec{\nabla}^2 \tilde{E} + \omega^2 \mu \epsilon \tilde{E} = 0$$

Consider x-polarized wave, propagating along $\hat{z}$

$$\frac{\partial^2}{\partial z^2} \tilde{E}_x + \omega^2 \mu \epsilon \tilde{E}_x = 0$$

$\tilde{E}_x = e^{-\gamma x}$ or $e^{\gamma x}$ ← what's γ?

Sub trial solution into wave eqn

$$(\gamma^2 + \omega^2 \mu E)\, \tilde{E}_x = 0$$

$$\gamma^2 = -\omega^2 \mu \epsilon$$

$$\gamma = j\beta, \quad \beta = \omega\sqrt{\mu\epsilon}$$

$$\tilde{E}_x(z) = e^{\mp j\beta z}$$

$$\tilde{H}_y(z) = \pm \frac{e^{\mp j\beta z}}{\eta}, \quad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

bnag angle

Now consider $\tilde{J} = \sigma \tilde{E} \neq 0$

Ampere's law: $\tilde{\nabla} \times \tilde{H} = \sigma \tilde{E} + j\omega\epsilon \tilde{E}$

$$= (\sigma + j\omega\epsilon)\tilde{E}$$

Replace every instance of $j\omega\epsilon$ w/ $(\sigma + j\omega\epsilon)$

$$\gamma^2 = -\omega^2\mu\epsilon = (j\omega\mu)(j\omega\epsilon) \Rightarrow \gamma = \sqrt{(j\omega\mu)(\sigma + j\omega\epsilon)}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

Wed 25 October Lecture 23

Lee
section X

① Perfect dielectric: $\sigma = 0$

② σ is very low, imperfect dielectric

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j/\omega}{j/\omega}\,\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{-\mu}{j\,\sigma/\omega - \epsilon}\cdot\frac{-1}{-1}} = \sqrt{\frac{\mu}{\epsilon(1 - j\frac{\sigma}{\omega\epsilon})}}$$

$$= \sqrt{\frac{\mu}{\epsilon}}\left(1 - j\frac{\sigma}{\omega\epsilon}\right)^{-1/2} = \sqrt{\frac{\mu}{\epsilon}}\left(1 + j\frac{\sigma}{2\omega\epsilon}\right)$$

$$\tau = \angle\eta = \tan^{-1}\left(\frac{\sigma}{2\omega\epsilon}\right) \text{ For } \frac{\sigma}{2\omega\epsilon} \ll 1$$

$$\approx \sigma/2\omega\epsilon, \quad |\eta| = \sqrt{\frac{\mu}{\epsilon}}$$

③ $\frac{\sigma}{\omega\epsilon} \gg 1$, good conductor

$$\gamma = j\omega\sqrt{\mu\epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right)} \quad \text{Neglect } 1$$

$$\approx j\omega\sqrt{-j\mu\sigma/\omega}$$

$$= \omega\sqrt{\frac{j^2}{j}\,\frac{\mu\sigma}{\omega}} = \sqrt{j\omega\mu\sigma}$$

$$j^{1/2} = \cos\frac{\pi}{4} + j\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}(1+j)\sqrt{\omega\mu\sigma} = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}} \quad \alpha + j\beta$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}\left(1 - j\frac{\sigma}{\omega\epsilon}\right)} = \sqrt{\frac{\mu}{-j\frac{\sigma}{\omega}}} = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\,e^{j\pi/4}, \quad \tau = \angle\eta = 45^\circ$$

④ Perfect conductor $\sigma \to \infty$, $\vec{E}_x = 0$

Propagation velocity

fiber cables — many frequencies together
multiplex and demultiplex them
diff velocities → dispersion
⇒ light waves

$V_p = \frac{\omega}{\beta}$ frequency dependent

$\lambda = \frac{2\pi}{\beta} = \frac{V_p}{f}$

Penetration depth $\delta \equiv \frac{1}{\alpha}$

Imperfect dielectric: $\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$

$$\delta = \frac{2}{\sigma\sqrt{\mu/\epsilon}} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

Make f very low to increase δ

e.g., submarine communication

Mon 30 Oct Lecture 25 Wave reflection

Lee
Section X

sin → $-j$

$\vec{E}_c = \cos(\omega t - \beta z)\hat{x} + \sin(\omega t - \beta z)\hat{y}$ in vacuum

What is average power density at $z>0$? $\frac{1}{2}\mathrm{Re}\{\tilde{E}_c \times \tilde{H}_c^*\}$

A) 0 B) $\frac{1}{2\eta_0}\hat{z}$ C) $-\frac{1}{2\eta_0}\hat{z}$ D) $\frac{1}{\eta_0}\hat{z}$ E) $-\frac{1}{\eta_0}\hat{z}$

$$\tilde{E}_c = e^{-\beta z j}\hat{x} + e^{j(\pi/2 - \beta z)}\hat{y}$$

$$\cancel{\tilde{H}_c = \frac{1}{\eta}[\cos(\omega t - \beta z)\hat{y} + \sin(\omega t - \beta z)(-\hat{x})]}$$

$$= \frac{1}{\eta_0}(\hat{y} + j\hat{x})e^{-j\beta z}$$

$$\cancel{\tilde{H}_c^* = \frac{1}{\eta_0}(\hat{y} - j\hat{x})e^{j\beta z}}$$

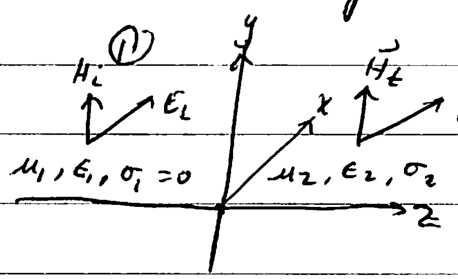
$$\tilde{E}_c = (\hat{x} - j\hat{y})e^{-j\beta z}$$

$$\cancel{\frac{1}{2}\mathrm{Re}\{\tilde{E}_c \times \tilde{H}_c^*\} = \frac{1}{2\eta_0}(\hat{z} + \hat{z}) = \frac{1}{\eta_0}\hat{z} \quad (D)}$$

Smith charts for impedance matching

Recall boundary condition equations:

$$\hat{n}\cdot(\vec{D}^+ - \vec{D}^-) = \rho_s$$
$$\hat{n}\cdot(\vec{B}^+ - \vec{B}^-) = 0$$
$$\hat{n}\times(\vec{E}^+ - \vec{E}^-) = \vec{0}$$
$$\hat{n}\times(\vec{H}^+ - \vec{H}^-) = \vec{J}_s$$



$z=0$ plane, no charge, no J_s

<u>Region 1</u>

$$\tilde{E}_i = \hat{x}E_0 e^{-j\beta_i z}, \quad \tilde{H}_i = \hat{y}\frac{E_0}{\eta_i}e^{-j\beta_1 z}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\sigma_1}}, \quad \beta_1 = \omega\sqrt{\mu_1\epsilon_1}$$

Cross into region 2, μ_2, ϵ_2 change, form of wave solutions changes due to change in σ

$$\gamma = j\omega\sqrt{\mu\epsilon} = j\beta \Rightarrow \gamma = \sqrt{(j\omega\mu_2)(\sigma_2 + j\omega\epsilon_2)} = \alpha + j\beta$$

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\epsilon_2}}$$

So, solutions in region 2 are

$$\tilde{E}_t = \hat{x}\tau E_o e^{-\gamma_2 z}, \quad \tilde{H}_t = \hat{y}\frac{\tau E_o}{\eta_2} e^{-\gamma_2 z}$$

$\tau \equiv$ transmission coefficient

In general, at boundary $z=0$

$$\tilde{E}_{ix} \neq \tilde{E}_{tx}$$
$$\tilde{H}_{ix} \neq \tilde{H}_{tx}$$

Add a reflected wave in region ① to enforce Maxwell's boundary conditions

Guess form of reflected wave

$$\tilde{E}_r = \hat{x}\Gamma E_o e^{j\beta_1 z} \quad \tilde{H}_r = -\hat{y}\frac{\Gamma}{\eta_1} e^{j\beta_1 z} E_o$$

$\Gamma \equiv$ reflection coefficient

<u>BCEs</u>

① Tangential $\tilde{E}$ continuous at $z=0$

$$\tilde{E}_{ix} + \tilde{E}_{rx} = \tilde{E}_{tx}$$
$$(1+\Gamma) E_o = \tau E_o$$
$$1+\Gamma = \tau$$

② Tangential $\tilde{H}$ continuous at $z=0$;

$$\tilde{H}_{iy} + \tilde{H}_{ry} = \tilde{H}_{ty}$$
$$(1-\Gamma)\frac{E_o}{\eta_1} = \tau\frac{E_o}{\eta_2}$$
$$1-\Gamma = \frac{\eta_1}{\eta_2}\tau$$
$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \quad \tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Special ~~case~~ cases

① Region 2 is perfect conductor, $\sigma_2 \to \infty$, $\eta_2 \to 0$, $\Gamma = -1$

"metal mirror" $\tau = 0$

falstad.com/emwave1/

Reflection At Conductor

② If $\eta_2 = \eta_1$, $\Gamma = 0$, $\tau = 1 \Rightarrow$ matched impedance,

③ Region 2, $\sigma_2 = 0$, but other parameters differ — no reflection

e.g. anti-reflecting coating on solar cell

blue-purple color

silicon dioxide/silicon nitride

"anti-reflection coating"

$\Downarrow$ can impedance match a single λ_0

ECE 329 1 November Lecture 26 Standing waves

Example from lecture 25

Plane wave in vacuum, $\tilde{E}_i = \hat{x}\sqrt{120\pi}\, e^{-j\beta z}\ \frac{V}{m}$

incident at $z=0$ on dielectric medium w/ $\mu=\mu_0$,

$$\epsilon = \frac{9}{4}\epsilon_0,\quad \eta_2 = \sqrt{\frac{\mu_0}{\frac{9}{4}\epsilon_0}} = \frac{2}{3}\eta_0$$

Solve $\langle \vec{\tilde{S}}_i \rangle$, $\langle \vec{\tilde{S}}_r \rangle$, $\langle \vec{S}_t \rangle$

~~Reflection coefficient~~ $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{2}{3}\eta_0 - \eta_0}{\frac{5}{3}\eta_0} = \boxed{-\frac{1}{5}}$

$$\tau = 1 + \Gamma = \boxed{\frac{4}{5}}$$

~~Reflected wave~~: $\tilde{E}_r = -\frac{1}{5}\hat{x}\sqrt{120\pi}\, e^{j\beta_1 z}$, $\tilde{H}_r = \frac{1}{5\eta_0}\hat{y}\sqrt{120\pi}\, e^{j\beta_1 z}$

$$\langle \vec{S}_r \rangle = \frac{1}{2}\mathrm{Re}\{\tilde{E}_r \times \tilde{H}_r^*\} = -\hat{z}\,\frac{1}{2}\,\frac{1}{25} = -\hat{z}\cdot\frac{1}{50}$$

Note $(\sqrt{120\pi})^2/\eta_0 = 1$

Transmitted wave: $\tilde{E}_t = \frac{4}{5}\hat{x}\sqrt{120\pi}\, e^{-j\beta_2 z}$,

$$\tilde{H}_t = \frac{4}{5\cdot\frac{2}{3}\eta_0}\hat{y}\sqrt{120\pi}\, e^{-j\beta_2 z}$$

$$\langle \vec{S}_t \rangle = \frac{1}{2}\mathrm{Re}\{\tilde{E}_t \times \tilde{H}_t^*\} = \hat{z}\cdot\frac{12}{25}$$

Incident wave: $\langle \vec{S}_i \rangle = \frac{1}{2}\mathrm{Re}\{\tilde{E}_i \times \tilde{H}_i^*\} = \hat{z}\cdot\frac{1}{2}$

$$|\langle \vec{S}_r \rangle| + |\langle \vec{S}_t \rangle| = |\langle \vec{S}_i \rangle|$$

Energy is conserved

Nano-
photonics
Plasmonics

Standing waves: plane wave incident on perfect conducting mirror at $z=0$, $\Gamma = -1 \Rightarrow$ standing wave

$\tilde{E}_i = \hat{x}E_0 e^{-j\beta_i z}$ $\quad \tilde{H}_i = \hat{y}\frac{E_0}{\eta_i}e^{-j\beta_i z}$

$\tilde{E}_r = -\hat{x}E_0 e^{j\beta_i z}$ $\quad \tilde{H}_r = \hat{y}\frac{E_0}{\eta_i}e^{j\beta_i z}$

$e^{jx} = \cos x + j\sin x$, $\sin(-x) = -\sin x$, $\cos(-x) = \cos x$

$\tilde{E} = \tilde{E}_i + \tilde{E}_r = \hat{x} E_0 (e^{-j\beta_1 z} - e^{j\beta_1 z})$

$= \hat{x} E_0 (\cos(\cancel{-}\beta_1 z) + \underbrace{j\sin(-\cancel{}\beta_1 z)}_{-j\sin(\beta_1 z)} - [\cos(\cancel{}\beta_1 z) + j\sin(\cancel{}\beta_1 z)])$

$= -2j\hat{x} E_0 \sin(\beta_1 z)$

$\tilde{H} = \tilde{H}_i + \tilde{H}_r = \hat{y} \frac{2E_0}{\eta_1} \cos(\beta_1 z)$

$\vec{E}(z,t) = \hat{x}\, 2E_0 \sin(\beta_1 z)(\sin \omega t)$, $\vec{H}(z,t) = \hat{y} \frac{2E_0}{\eta_1} \cos(\beta_1 z) \cos(\omega t)$

position part and ~~si~~ time part are now separate

$\vec{E}$ locked to 0 for all times for $z=0$, $\vec{H}$ node stuck at $\frac{\pi}{2\beta}$, varies in t for $z=0$

unlike d'Alembert solutions where time and z together in 1 term

Nodes are called shorts ($\tilde{v}$oltage $= 0$)

time-varying sheet current at $z=0$

$\tilde{E} = -2j\hat{x}E_0 \sin(\beta_1 z)$, $\tilde{H} = \hat{y} \frac{2E_0}{\eta_1} \cos(\beta_1 z)$

$\frac{1}{2} Re\{\tilde{E} \times \tilde{H}^*\} = \frac{1}{2} Re\{-2j\hat{x} E_0 \sin(\beta_1 z) \times \hat{y} \frac{2E_0}{\eta_1} \cos(\beta_1 z)\}$

$= 0$ (pure imaginary = pure real × pure imaginary)

standing waves carry no net energy

Notes on standing wave forms: nodes separated by $\frac{\lambda}{2}$

Nodes of $\vec{E}$ and $\vec{H}$ separated by $\frac{\lambda}{4}$

$\vec{H}(0,t) = \hat{y} \frac{2E_0}{\eta_1} \cos(\omega t)$ at $z=0$

Apply boundary condition equations: $\vec{J}_s = \hat{x} \frac{2E_0}{\eta_1} \cos(\omega t)\ \frac{A}{m}$

Real metal: "skin depth"

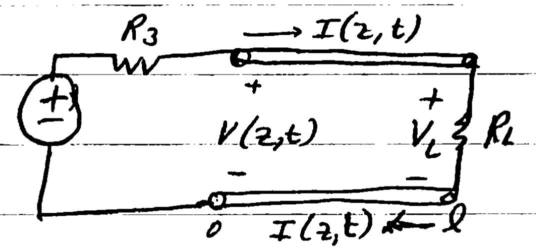
Lee
Sec X

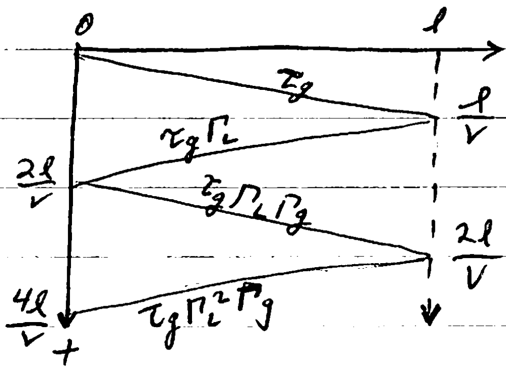
Wed 8 November Lecture 29

Bounce diagrams and examples

From last time:
$$\Gamma_\ell = \frac{R_L - Z_0}{R_L + Z_0}$$

$$\Gamma_s = \frac{R_s - Z_0}{R_s + Z_0}$$





forward propagation

$$V(z,t) = \tau_g \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \, \delta\left(t - \frac{z}{v} - n\frac{2\ell}{v}\right)$$

backward propagation

$$+ \tau_s \Gamma_\ell \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \, \delta\left(t + \frac{z}{v} - (n+1)\frac{2\ell}{v}\right)$$

$$I(z,t) = \frac{\tau_g}{Z_0} \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \, \delta\left(t - \frac{z}{v} - n\frac{2\ell}{v}\right)$$

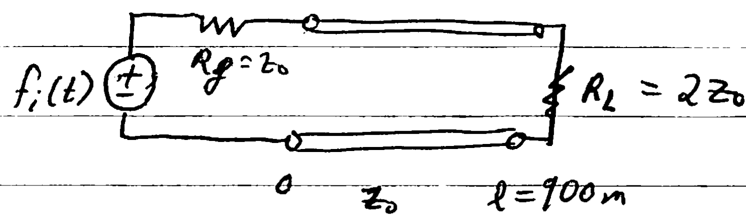
$$- \frac{\tau_g R_L}{Z_0} \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \, \delta\left(t + \frac{z}{v} - (n+1)\frac{2\ell}{v}\right)$$

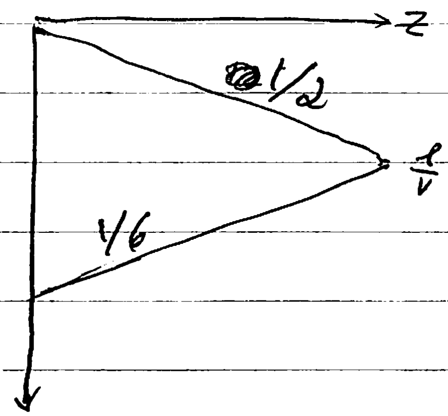
Note that $|\Gamma_L, \Gamma_g| \le 1$, so $(\Gamma_L \Gamma_g)^n$ rapidly shrinks as n increase, i.e., approaches steady state limit

Example 1, TL $l = 900\,m$, $v = c$, $R_L = 2Z_0$,

$f_i(t) = \sin(\omega t)\, u(t)$

$R_g = Z_0$, $\frac{\omega}{2\pi} = 1\,MHz$





Determine all coefficients

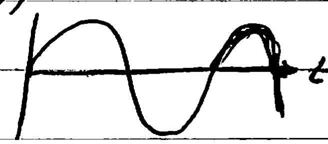
$$\tau_s = \frac{Z_0}{R_s + Z_0} = \frac{1}{2}$$

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = \frac{2Z_0 - Z_0}{2Z_0 + Z_0} = \frac{1}{3}, \quad \Gamma_G = \frac{R_g - Z_0}{R_g + Z_0} = 0, \quad \frac{2l}{v} = 6\mu s$$

Impulse response function, $h_z(t) = \frac{1}{2}\delta(t - \frac{z}{c}) + \frac{1}{6}\delta(t + \frac{z}{c} - 6\mu s)$

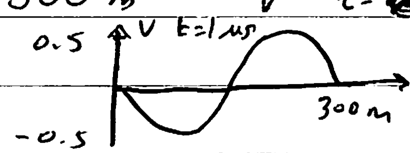
$$V(z,t) = \frac{1}{2}\sin\omega(t - \tfrac{z}{v})u(t - \tfrac{z}{v}) + \frac{1}{6}\sin\omega(t + \tfrac{z}{v} - 6)u(t + \tfrac{z}{v} - 6)$$

Sketch $v(z)$ at $t = 1\mu s$



How far has leading edge travelled?

300 m   V $t = 1\mu s$



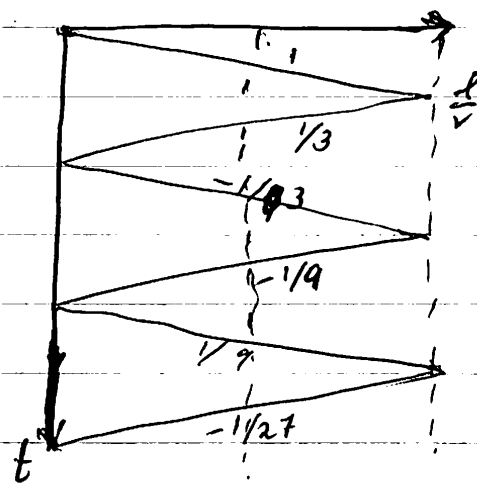
Example 2: $Z_0 = 50\,\Omega$, $v = c$, $l = 2400$ m, $R_s = 0$, $R_L = 100\,\Omega$

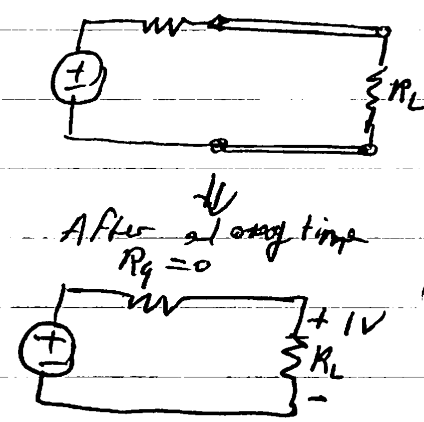
$f_i(t) = u(t)$ Plot: $V(1200, t)$

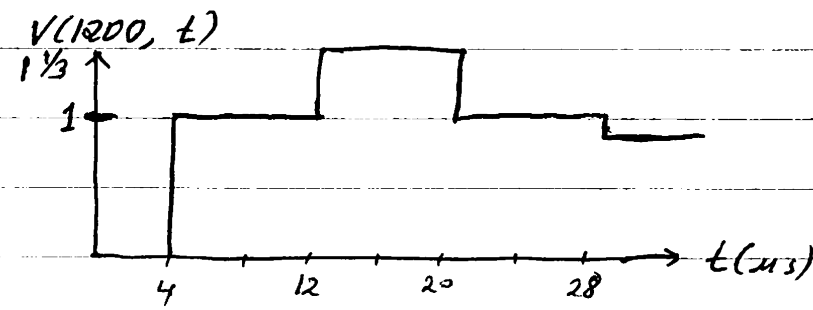
$$T_g = \frac{Z_0}{R_g + Z_0} = 1, \quad \Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = -1, \quad \Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = \frac{1}{3}$$

round-trip time $= \frac{2l}{v} = \frac{4800\text{ m}}{300\text{ m}/\mu s} = 16\,\mu s$

transit time $= \frac{l}{v} = 8\,\mu s$







$$V(1200, t) = u(t-4) + \frac{1}{3}u(t-12) - \frac{1}{3}u(t-20) - \frac{1}{9}u(t-28) + \cdots$$

$t \to \infty$, $V(1200, t) \to 1$ V = DC component

radiofrequency plasma

$\omega = 0$, $\lambda \to \infty$

$l \ll \lambda$, TL is just a lumped circuit

Sat 11 Nov HKN Review Session

Two sets of slides — Sp17, Fa17

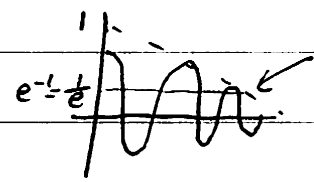
Lectures 20-30, go thru Fa16 exam

Poynting Thm & Poynting Flux $\vec{S} = \vec{E} \times \vec{H}$

Phasors — sin $\rightarrow -j$

Wave propagation ~~propagation~~ eqns → note card

Propagation in Various Media $\gamma = \alpha + j\beta$



comes from

α, $e^{-\alpha}$ $\quad e^{j(\alpha + j\beta)}$

α — attenuation, β — propagation const.

Gets table of fields in various media on exam

$\frac{d}{dt} \rightarrow j\omega$ $\quad \vec{\nabla} \cdot \tilde{E} = 0 \quad \vec{\nabla} \times \tilde{E} = -j\omega\mu \tilde{H}$

$\quad \vec{\nabla} \cdot \tilde{H} = 0 \quad \vec{\nabla} \times \tilde{H} = j\omega\epsilon \tilde{E}$

Skin depth $1/\alpha$

Wave Polarization

Circular polarization

$\vec{E} = E_1 \cos(\omega t - \beta z) + E_2 \sin(\omega t - \beta z)$

1) $|E_1| = |E_2|$ $\quad E_1 = -2 \quad E_1 = 2$ $\quad$ cos even fxn

2) 90° phase difference $\quad E_2 = 2 \quad E_2 = -2$ $\quad$ sin odd fxn

3) Same β $\hat{r}$ direction $\quad$ depends on wave toward/away from you

right circular or left circular $\quad$ (CW/CCW)

RHR lead to lag thumb in propagation direction
dir dir

RHR: $\underbrace{\text{Lead} \times \text{Lag}}$ = Propagation $\quad$ cos leads sin

at any time e.g. 0 $\quad$ −sin leads cos

Reflection and Transmission

Boundary conditions

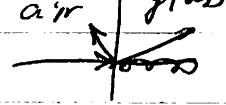
$$\Gamma = \frac{\bar{E}_1^-}{\bar{E}_1^+} = \frac{\bar{\eta}_2 - \bar{\eta}_1}{\bar{\eta}_1 + \bar{\eta}_2}$$

1 – medium coming from
2 – medium going into

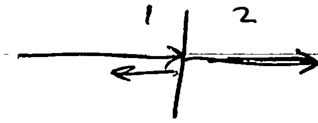
$$\tau = 1 + \Gamma = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{\bar{E}_2^+}{\bar{E}_1^+}$$

τ transmission — coefficients for E; careful for H, include correct η

air | glass



1 | 2



$\gamma = -1 \Rightarrow$ mirror

Transmission lines

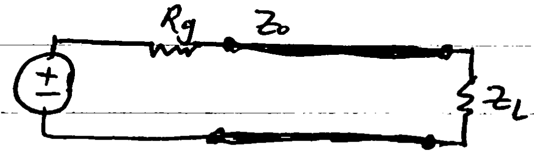
Telegrapher's eqns — write down geometric factors on note sheet

$$-\frac{\partial V}{\partial z} = \mathcal{L}\frac{\partial I}{\partial t}$$

$$-\frac{\partial I}{\partial z} = \mathcal{C}\frac{\partial V}{\partial t}$$

$\mathcal{C} = \epsilon GF$, $\mathcal{L} = \mu/GF$

$\tau_g = \frac{Z_0}{R_g + Z_0}$ injection coefficient, amount to make into transmission line



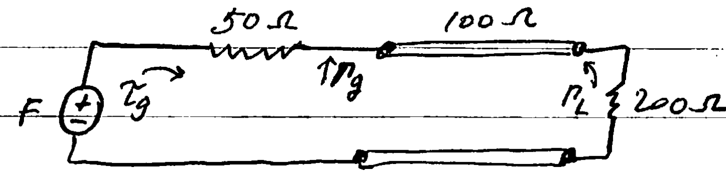
$F \to \tau_g F \to \tau_g \Gamma_L F \to \tau_g \Gamma_L \Gamma_G F$

Not inf/∞ sum, will ask 2, 3 bounces or after some T

Partial bounce doesn't count because hasn't reached point

Fa16 Exam

4.



(a)

$$\tau_g = \frac{Z_0}{Z_0 + R_g} = \frac{100}{100+50} = \boxed{\frac{2}{3}} = \tau_g$$

no units
coefficients for voltages

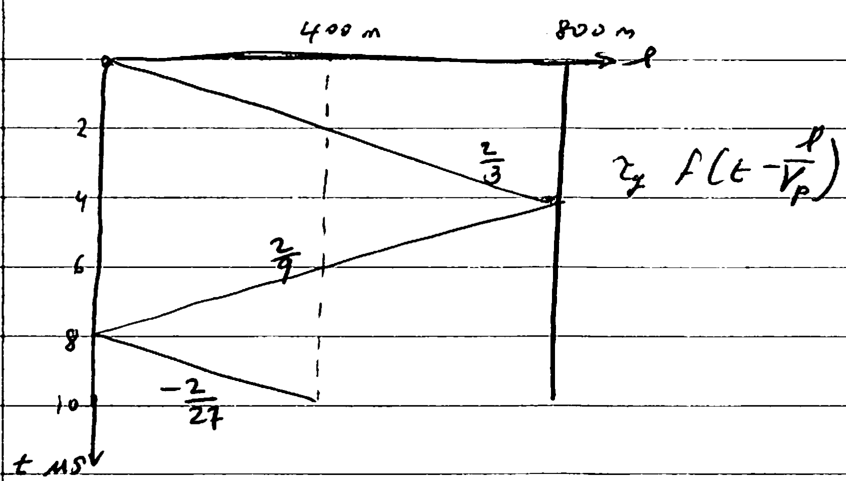
$$\Gamma_g = \frac{-100+50}{100+50} = \frac{-50}{150} = \boxed{\frac{-1}{3} = \Gamma_g}$$

$\tau_g = 1 + \Gamma_g$

$$\Gamma_L = \frac{200-100}{200+100} = \boxed{\frac{1}{3} = \Gamma_L}$$

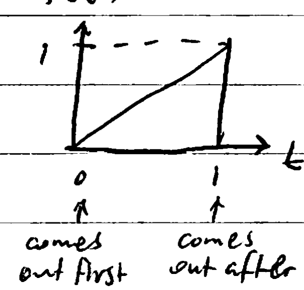
(b) $200\ m/\mu s = v_p$

$T = 4\ \mu s = 800m / 200\ m/\mu s$



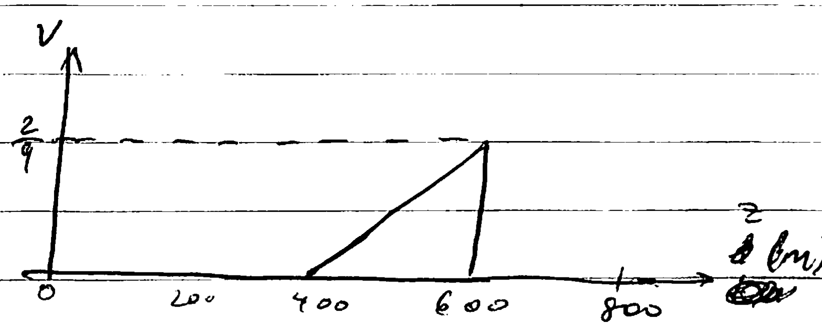
$\tau_g\, f(t - \frac{\ell}{v_p})$

(c)

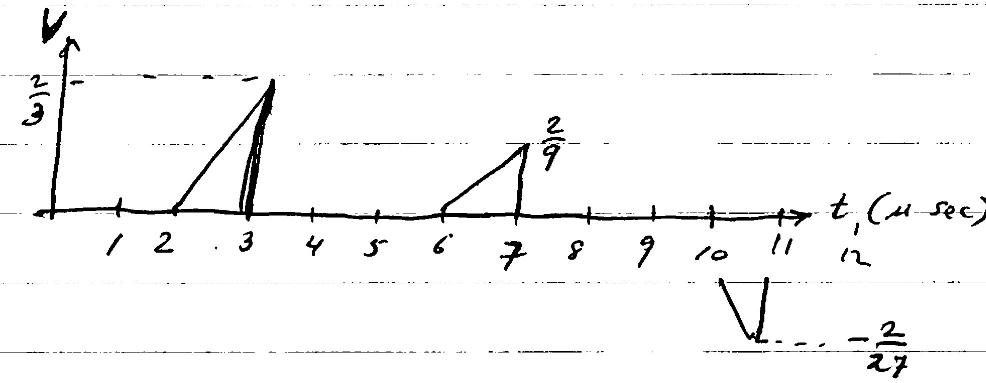


$6\ \mu s\,(200\ m/\mu s) = 1200\ m$

↳ 400 m back



(d)



1. (a) $\hat{S} = -\hat{z}$ $\qquad \hat{y}\times\hat{x} = -\hat{z}$

$\hat{H} = \hat{x}$

(b) $V_p = \frac{\omega}{\beta} = \frac{2\pi\times10^{14}}{\pi\times10^{6}} = \boxed{2\times10^{8}\ m/s}$

(c) $V = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}} = \frac{2}{3}c$

$\left(\frac{2}{3}\right)^2 = \frac{1}{\mu_r\varepsilon_r} \Rightarrow \boxed{\varepsilon_r = \frac{9}{4}}$

$\eta = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{2}{3}\eta_0 \Rightarrow \boxed{\eta = 80\pi\ \Omega}$

(d) $\tilde{E} = \boxed{4e^{+j\beta z}\,\hat{y}\ \frac{V}{m}}$

(e) $\tilde{E} = \eta\tilde{H} \Rightarrow \tilde{H} = \tilde{E}/\eta = \boxed{\frac{4}{\eta}e^{j\beta z}\,\hat{x}\ \frac{A}{m}}$

(f) $\tilde{E}^{+} = 4e^{-j\beta z}\,\hat{y}\ \frac{V}{m}$

(stays dir on ~~so~~ two sides of sheet)

(g) $\tilde{H}^{+} = \frac{4}{\eta_0}e^{-j\beta z}(-\hat{x})\ \frac{A}{m}$ (changes dir on two sides of sheet)

(h) $|H| = \frac{x}{2}$ $\quad x \quad$ $|H| = \frac{x}{2}$

i) $\tilde{J}_s = -\frac{8}{\eta}\hat{y}\ \frac{A}{m}$ must have to

ii) $\tilde{J}_s = -\frac{8}{\eta_0}\hat{y}\ \frac{A}{m}$ do w/ both η, η_0

in vacuum

$\hat{J} = -\hat{E}$ $\longrightarrow$ iii) $\tilde{J}_s = -4\left(\frac{1}{\eta_0}+\frac{1}{\eta}\right)\hat{y}\ \frac{A}{m}$

iV) $\tilde{J}_s = 4\left(\frac{1}{\eta_0}+\frac{1}{\eta}\right)\hat{y}\ \frac{A}{m}$

$\hat{n}\times(H^{+}-H^{-}) = J_s$

2. a) 1. Magnitudes are the same

2. β is the same

3. $\frac{\pi}{2}$ out of phase

$|A| = 3$

$B = -20\pi$

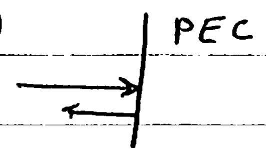
$|C| = \frac{\pi}{2}$

$\hat{z} \times -\hat{y} = \hat{x}$

$A = -3$ or $A = 3$

$C = -\frac{\pi}{2}$ $\quad C = \frac{\pi}{2}$

b)



$$\Gamma = \frac{0 - \eta}{0 + \eta} = -1$$

$$\vec{E}_r = -3\cos(2\pi \times 10^9 t + 20\pi x)\,\hat{z} + 3\cos(2\pi \times 10^9 t + 20\pi x - \tfrac{\pi}{2})\,\hat{y}\ \frac{V}{m}$$

c) $\hat{y} \times \hat{z} = \hat{x} \Rightarrow \boxed{LCP}$

d) $\sigma = 100 \quad \varepsilon = 2\varepsilon_0 \quad \mu = \mu_0 \quad f = 1 \times 10^9$

$$\frac{\sigma}{\omega\epsilon} = \frac{100}{2\pi \times 10^9 \,(2 \times 8.85 \times 10^{-12})} = \frac{100 \times 10^3}{2\pi \times 2 \times 8.85} >> 1$$

Good conductor

$\alpha = \sqrt{\pi f \mu \sigma}$

$$\delta = \frac{1}{\alpha} = \boxed{\frac{1}{\sqrt{\pi f \sigma \mu}}\ m}$$

$\sigma = 100$

$\mu = \mu_0$

$f = 1 \times 10^9$

3. a) $\dfrac{Z_{eq}}{R_g + Z_{eq}} \Rightarrow Z_{eq} = 100 // 200 = \dfrac{200}{3}$

$$\frac{200/3}{100 + 200/3} = \boxed{\frac{2}{5} = \tau_g}$$

$V(0^+ = t,\ 0^+ = z)$

$$(10)(\tau_g) = 10\left(\frac{2}{5}\right) = \boxed{4V}$$

$$V(0^+ = t,\ 0^- = z) = \boxed{4V}$$

$$I(0^+ = t,\ 0^+ = z) = V/Z_2 = \frac{4V}{200\,\Omega} = \boxed{20\ mA}$$

$$I(0^+ = t,\ 0^- = z) = V/Z_1 = \frac{4V}{100\,\Omega} = \boxed{40\ mA}$$

b) $\dfrac{Z_{going\ into} - Z_{coming\ from}}{\text{'' } + \text{ ''}}$

$$Z_{EQ} = R_g // Z_1 = 50\ \Omega$$

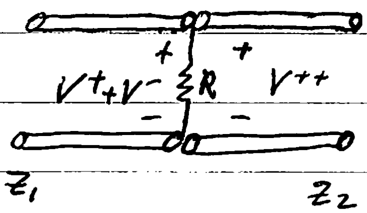
$$\Gamma_+ = \frac{50 - 200}{250} = \frac{-150}{250} = \boxed{-\frac{3}{5}}$$

c) $Z_{EQ} = R_g // Z_1 = 200/3\ \Omega$

$$\Gamma_- = \frac{\frac{200}{3} - 100}{\frac{200}{3} + \frac{100}{3}} = \boxed{-\frac{1}{5}}$$

Lee
Section X

Mon 13 Nov lecture 31



What is Γ_{12}, τ_{12}?

Voltage wave from left

$V^+(t - \frac{z}{v_1})$

$V^-(t + \frac{z}{v_1})$ reflected

$V^{++}(t - \frac{z}{v_2})$ transmitted to line 2

KVL: $V^+ + V^- = V^{++}$

KCL: $\frac{V^+ - V^-}{Z_1} = \frac{V^{++}}{R} + \frac{V^{++}}{Z_2}$

$$V^+ - V^- = Z_1\left(\frac{Z_2 V^{++}}{Z_2 R} + \frac{R V^{++}}{Z_2 R}\right)$$

$$= Z_1\left(\frac{Z_2 + R}{Z_2 R}\right) V^{++}$$

$$V^+ - V^- = \frac{Z_1}{Z_{eq}} V^+$$

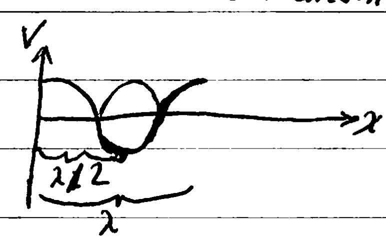
$$Z_{eq} = \frac{Z_2 R}{Z_2 + R}$$

parallel combo of Z_2 and R

$$\Gamma_{12} = \frac{V^-}{V^+} = \frac{Z_{eq} - Z_1}{Z_{eq} + Z_1}$$

$$\tau_{12} = \frac{V^{++}}{V^+} = 1 + \Gamma_{12} = \frac{2 Z_{eq}}{Z_{eq} + Z_1}$$

Resonances on transmission lines

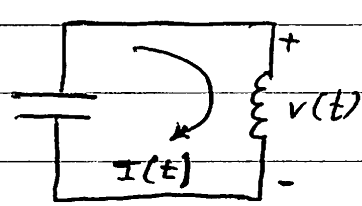


When looking at absolute value,

3 peaks (2 full cycles) ⇒ one period

2 peaks ⇒ 1/2 period

peak → trough ⇒ 1/4 period



TLs can have an infinite number of resonances

↳ Fourier sums of resonances

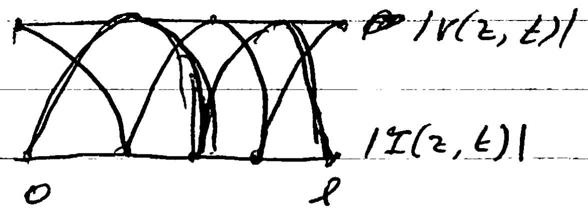
quality factor

opera singer breaking glass

glass rim ringing

music frequencies thru floor of room

telegrapher's eqns



Basic boundary condition is that $I(z,t) \to 0$ at $z=0, z=\ell$

"stub" open-circuit on both ends

$$I(z,t) = \frac{f(t-z/v)}{Z_0} - \frac{g(t+z/v)}{Z_0}$$

$$I(0,t) = \frac{f(t)}{Z_0} - \frac{g(t)}{Z_0}$$

$$I(\ell,t) = \frac{f(t-\ell/v)}{Z_0} - \frac{g(t+\ell/v)}{Z_0} = 0$$

$$\downarrow$$

$$g(t) = f(t)$$

$$\downarrow$$

periodic $\quad f\left(t-\frac{z}{v}\right) = f\left(t+\frac{z}{v}\right)$

$$T = \frac{2\ell}{v} \rightarrow f(t) = f\left(t+\frac{2\ell}{v}\right)$$

$$\omega_0 = \frac{2\pi}{T} = \frac{\pi v}{L}$$

$\downarrow$

~~[illegible]~~ Fundamental frequency

$$f(t) = F_0 + \sum_{n=1}^{\infty} F_n \cos(n\omega_0 t + \theta_n)$$

$\downarrow$ "DC offset"

↳ harmonically related frequencies

F_n, θ_n = Fourier coefficients

Sub in $\left(t-\frac{z}{v}\right)$ for t...

$$I(z,t) = \sum_{n=1}^{\infty} \frac{F_n}{Z_0}\left[\cos(n\omega_0 t + \theta_n - n\beta_0 z) - \cos(n\omega_0 t + \theta_n + n\beta_0 z)\right]$$

$\beta_0 \equiv \frac{\omega_0}{v} = \frac{\pi}{\ell}$ wavenumber | node in current $\longleftrightarrow$ antinode in voltage

" voltage $\longleftrightarrow$ " current

$$\tilde{I}(z) = \sum_{n=1}^{\infty} \frac{F_n}{Z_0} e^{j\theta n} \left[e^{-jn\beta_0 z} - e^{jn\beta_0 z} \right]$$

$\cos\theta = \cos(-\theta)$

$\sin(-\theta) = -\sin\theta$

Review hw 11

TA Review Session MT III

Lectures 20 - 30

Section X - 201 ECED

<u>Lectures 20 - 21</u>

Poynting thm, phasor form

<u>Lectures 22-23</u>

Wave propagation in media

Wave polarization

<u>Lec 25-26</u>

Wave reflection and transmission

$$\Gamma_{12} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}$$

perfect conductor

$\Gamma = -1 \quad \tau = 0$

$$\tau_{12} = 1 + \Gamma_{12} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

matched impedance

$\Gamma = 0 \quad \tau = 1$

<u>Lectures 27-30</u>

Transmission lines

Transmission & reflection coefficients

Bounce diagrams

Multi-line circuits (parallel/series in lumped circuits)

#4 Fa16

#3 Sp 17

$\gamma \eta = j\omega\mu$

$\gamma / \eta = \sigma + j\omega\epsilon$

$\vec{E} = 3e^{-3x}\cos(3\pi 10^6 t - 4x + 60^\circ)\hat{z}\ \frac{V}{m}$

$\alpha = 3,\ \beta = 4,\ \gamma = \alpha + j\beta = 3 + j4 = \begin{cases} |\gamma| = \sqrt{3^2 + 4^2} = 5 \\ \angle\gamma = 53^\circ \text{ given} \end{cases}$

For note card

general form for damped cosine

A, ω, θ, α, β

constants: $\mu_0, \epsilon_0, \eta_0, c$, etc.

units

Table of ~~dielect~~ media + propagation

$$\lambda = \frac{2\pi}{\beta} = \frac{\pi}{2}$$

$$8\pi 10^6 = 2\pi f \Rightarrow f = 4\,MHz$$

$$v_p = \frac{\omega}{\beta} = 2\pi \times 10^6$$

$$\eta = \frac{j\omega\mu}{\gamma}$$

$$\gamma = 3 + j4$$

amplitude

$$\eta = \frac{3\omega\mu}{3+j4} = \frac{32\pi^2 \angle 90^\circ}{5 \angle 53}$$ phase (subtract)

$$= \frac{32}{50} \angle 90-53 = 0.64\pi^2 \angle 37^\circ$$

$$\begin{cases} \hat{u}_p = \hat{x} \\ \tilde{E} \end{cases}$$

$$\hat{H} = \frac{\overbrace{\hat{a}_p}^{\hat{x}} \times \overbrace{\tilde{E}}^{\hat{z}}}{\eta} = \frac{3(-\hat{y})}{0.64\pi^2} \cos(8\pi \times 10^6 \ldots + 60^\circ - 37^\circ)$$

$$\left.\begin{aligned} \tilde{E} &= 3e^{-3x} e^{-j4x} e^{60^\circ j} \hat{z} \\ \eta &= |\eta| \measuredangle \eta = |\eta| e^{j\measuredangle\eta} \end{aligned}\right\} \Rightarrow \tilde{H} = \frac{\hat{a}_p \times E}{\eta} = (-\hat{y}) 3e^{-3x} e^{-j4x} \frac{e^{60j}}{0.64\pi^2 e^{j3\cdots}}$$

$$\hat{H} = Re\{H e^{j\omega t}\}$$

→ Fa15 #2 Good conductor? Imperfect dielectric?

Part (c) — wave decay

circular polarization — lead & lag

$\cos^2(\omega t - \beta x)$

$\frac{1}{2}[1 + 2\cos(2\omega t \ldots 2\beta x)]$

HW 9 #1 — Poynting thm, instantaneous power

$\vec{E} = 10\cos(\omega t - \beta x)\hat{y}$, $H = \frac{10}{\eta_0}\cos(\omega t - \beta x)\hat{z}$ $\quad \langle P \rangle = \frac{1}{2} Re\{\vec{E} \times \vec{H}^*\}$

$\tilde{E} = 10 e^{-j\beta x}$, $\tilde{H} = 10/\eta_0 \, e^{-j\beta x}\hat{z}$ $\quad \langle E \times H \rangle = \frac{100}{\eta_0}\cos^2(\omega t - \beta x)$

Sp 17 #2 (i) ~~(a)~~ α is small

(a) $e^{\alpha y}$ - decays much fast by table values

$\frac{1}{e}$ value

Sp 17 #1 a) Good conductor

$$\frac{\sigma}{w\epsilon} \gg 1$$

$$\frac{\sigma}{2\pi f \epsilon} \gg 1$$

$\rightarrow$ b) Maxwell's eqn

$\nabla \times E$

$\nabla \times B$

J same as E?

diagonal linear propagation

No resonance circuits on exam

$\tau + 1 = \Gamma$? always

Pollet & series multi-line

Wed 15 Nov Lecture ~~MT~~ MT Review section X Lee

Sp 17 #1 (i)

(a) $\delta \equiv \frac{1}{\alpha}$, $\lambda = \frac{2\pi}{\beta}$ $\Rightarrow \delta = \frac{\lambda}{2\pi}$ X less than one wavelength

(b) $V_p = \frac{\omega}{\beta}$ frequency dependent in not free space

(iii) $\frac{|\vec{E}|}{|\vec{H}|} = \eta$

(iv) $\epsilon = 4\epsilon_0$, $\mu = \mu_0$, $\sigma = 400\,\omega\epsilon_0$

$\frac{\sigma}{\omega\epsilon_0} = 100 \Rightarrow |\eta_1| \Rightarrow \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ $\quad \eta_1 << \eta_2 \Rightarrow \Gamma = 1$

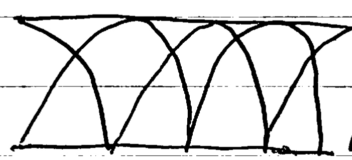
(v) $\tau = \frac{Z_0}{R_g + Z_0} = \frac{50}{R_g + 50}$ $\quad \tau \cdot 30 = 25 \Rightarrow \tau = \frac{5}{6}$ $\quad R_g = 10\,\Omega$

(vi) Reflection coefficient $= 0$

Section X
Lee

Fri 17 Nov lecture 32

$e^{jx} = \cos x + j\sin x$
$\cos\theta = \cos(-\theta)$, $\sin(-\theta) = -\sin\theta$



Open-ended stub

$$\tilde{I}(z) = \sum_{n=1}^{\infty} \frac{F_n}{Z_0} e^{j\theta_n}\left[e^{-jn\beta_0 z} - e^{jn\beta_0 z}\right]$$

$$= \sum_{n=1}^{\infty} \frac{F_n}{Z_0} e^{j\theta_n}(-2j)\sin(n\beta_0 z)$$

$$I(z,t) = \mathrm{Re}\{\tilde{I}(z)\cdot e^{j\omega t}\}$$

$$\sum = \mathrm{Re}\{-2j \cdot e^{j(n\omega_0 t + \theta_n)}\sin(n\beta_0 z)\}$$

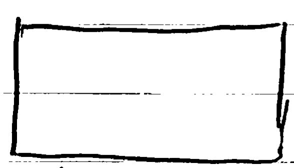
$$\sum = \mathrm{Re}\{-2j\cos(n\omega_0 t + \theta_n) - 2j^2\sin(n\omega_0 n + \theta_n)\}$$

$$= \sum_{n=1}^{\infty} \frac{2F_n}{Z_0}\sin(n\omega_0 t + \theta_n)\sin(n\beta_0 z)$$

$$\tilde{V}(z) = \sum_{n=1}^{\infty} F_n e^{j\theta_n}\left[e^{-jn\beta_0 z} + e^{jn\beta_0 z}\right]$$

$$V(z,t) = \sum_{n=1}^{\infty} 2F_n\cos(n\omega_0 t + \theta_n)\cos(n\beta_0 z)$$

- Open-ended stub hosts standing waves with
  $\omega = \frac{\pi v}{l} n$ $\frac{rad}{s}$ or $f = \frac{v}{2l} n$ Hz or $\lambda = \frac{v}{f} = \frac{2l}{n}$ or $l = n\frac{\lambda}{2}$
- Line length is an integer multiple of $\lambda/2$ for each resonance



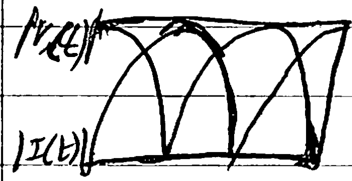
← same, but $V(z,t)$, $I(z,t)$ are swapped

"Shorted stub"

What if TL is shorted at $z=l$, open at $z=0$?

→ blended BCs: $V(0,t)$ = anti-node, $I(0,t) = 0$
$V(l,t) = 0$, $I(l=t)$ = anti-node

→ l = odd multiples of $\lambda/4$



Nulls of $\cos(\beta z)$ and $\sin(\beta z)$ are separated by odd multiples $\frac{\lambda}{4}$

$$\frac{\lambda}{4} = \frac{2\pi/\beta}{4} = \frac{\pi}{2\beta}$$

For resonance, $l = \frac{\lambda}{4}(2n+1)$, $n \geq 0$

$f\lambda = v$ to get resonant freq.

$$f = \frac{v}{2l}\left(\frac{1}{2}+n\right), \quad \omega = \frac{\pi v}{l}\left(\frac{1}{2}+n\right)$$

<u>Example</u>: Lossless TL, $Z_0 = 50\Omega$, $v = c$

$l = 600\text{m}$, open at $z=0$, short at $z=l$

a) Resonant frequencies b) Resonant current modes

c) Res voltage modes

a) $$600\text{m} = \frac{\lambda}{4}(2n+1) = \frac{c/f}{4}(2n+1)$$

$$f = \frac{c}{4}(2n+1)\frac{1}{600\text{m}} = \frac{1}{8\mu s}(2n+1) = \frac{1}{8}\text{MHz}\,(2n+1), \quad n \geq 0$$

b) current modes vanish at $z=0 \rightarrow$ use sin

$\sin(\beta z)\sin(\omega t)$

$\omega = 2\pi f = (2n+1)\frac{\pi}{4}$ Mrad/s

$\beta = \frac{2\pi}{\lambda = c/f} = (2n+1)\frac{\pi}{1200}$ Mrad/s

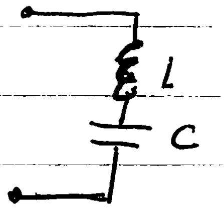
$$I_n(z,t) = \sin\left((2n+1)\frac{\pi}{1200}z\right)\sin\left((2n+1)\frac{\pi}{4}t\right)$$

c) $$-\frac{\partial V}{\partial z} = \mathcal{L}\frac{\partial I}{\partial t}$$

$$\frac{\partial V}{\partial z} = -\mathcal{L}(2n+1)\frac{\pi}{4}\sin\left((2n+1)\frac{\pi}{1200}z\right)\cos\left((2n+1)\frac{\pi}{4}t\right)$$

32 Input impedance and microwave resonators

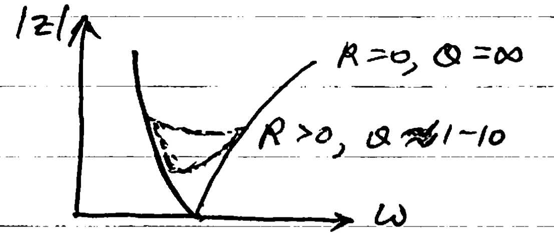
Recall ECE 210

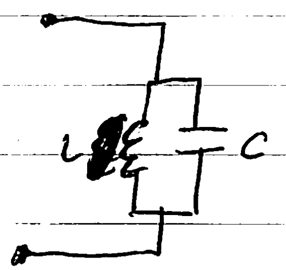


Series resonant circuit

$$Z_s = j\omega L + \frac{1}{j\omega C} = j\left(\omega L - \frac{1}{\omega C}\right)$$

$$@\ \omega_r = \frac{1}{\sqrt{LC}} \qquad Z_s = j\left(\frac{\sqrt{L}}{\sqrt{C}} - \frac{\sqrt{LC}}{C}\right) = 0$$

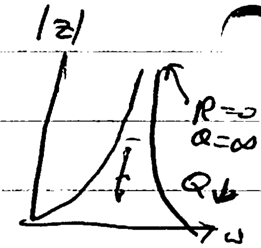




Parallel resonant circuit

$$Z_p = \frac{j\omega L \cdot \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1-\omega^2 LC}$$

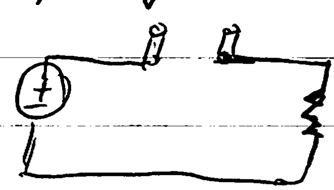
$$@\ \omega_r = \frac{1}{\sqrt{LC}}, \quad Z_p = \infty$$



admittance $Y = \frac{1}{Z}$ $\quad Y_p = j\left(\omega C - \frac{1}{\omega L}\right)$, $@\ \omega_r,\ Y_p = 0.$

- Use a TL stub as a ~~at~~ circuit component at high frequency, ← b/c of DC limit (open/short of circuits)

f(t)



reactance

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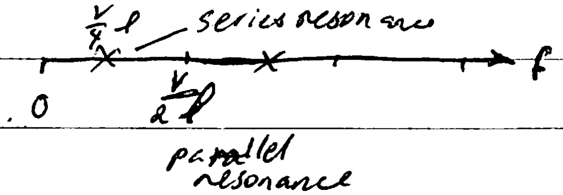
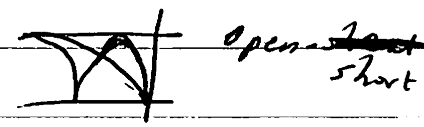
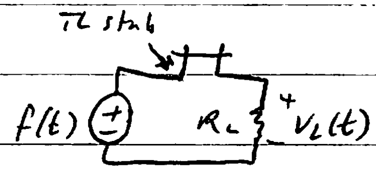
Section X

Mon 27 Nov Lecture 33 Input Impedance







At series resonances $\Rightarrow$ short, entire input signal appears across R_L

parallel resonances $\Rightarrow$ open, R_L see nothing

Away from resonance

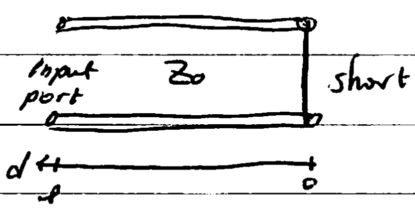
$$V(z,t) = Re\{V^+ e^{j\omega(t - z/v)}\} + Re\{V^- e^{j\omega(t+z/v)}\}$$

$$\Leftrightarrow \tilde{V}(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$$

$$I(z,t) = \left(Re\{V^+ e^{j\omega(t-z/v)}\} - Re\{V^- e^{j\omega(t+z/v)}\}\right) / Z_0$$

$$\Leftrightarrow \tilde{I}(z) = \frac{V^+ e^{-j\beta z} - V^- e^{j\beta z}}{Z_0}$$

Shift coordinate systems



$$z(\ell) = \frac{\tilde{V}(d-\ell)}{\tilde{I}(d-\ell)}$$

$$\tilde{V}(d) = V^+ e^{j\beta d} + V^- e^{-j\beta d}$$

$$\tilde{I}(d) = \frac{V^+ e^{j\beta z} - V^- e^{-j\beta d}}{Z_0}$$

Apply voltage BC at short termination:

$$V(0,t) = 0 \Leftrightarrow \tilde{V}(0) = V^+ + V^- = 0, \quad V^- = -V^+$$

$$\tilde{V}(d) = V^+(e^{j\beta d} - e^{-j\beta d})$$

$$= j2\, V^+ \sin(\beta \cdot d)$$

$$\tilde{I}(d) = \frac{V^+(e^{j\beta d} + e^{-j\beta d})}{Z_0 = \frac{1}{Y_0}} = Y_0\, 2V^+ \cos(\beta d)$$

Input impedance, $Z(d=l) = \frac{\tilde{V}(l)}{\tilde{I}(l)} = jZ_0 \tan(\beta l)$

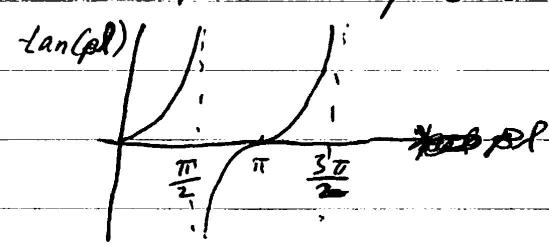
Recall: $Z = \underset{\text{resistive}}{R} + j\underset{\text{reactive}}{X}$ negative reactance = capacitive

positive reactance = inductive

$Z = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C}$

Observations

① Input impedance is pure reactive/imaginary for all l



$\beta l = \frac{w}{v} l = \frac{2\pi f}{v} l = \frac{2\pi}{\lambda} l$

② When $\frac{2\pi}{\lambda} l < \frac{\pi}{2}$, tan is positive and Z is inductive

$l < \frac{\lambda}{4}$ = quarter wavelength

③ When $\frac{\pi}{2} < \beta l < \pi$, tan is negative, Z capacitive

$\frac{\lambda}{4} < l < \frac{\lambda}{2}$

④ Reactance is periodic, all possible impedances can be derived for shorted TL stub of $0 < l < \frac{\lambda}{4}$

For $l << \frac{\lambda}{4}$, pure inductance

$Z(l) = jZ_0 \tan(\beta l) \approx \beta l$

$= jZ_0 \beta l$

$= j\sqrt{\frac{\mathcal{L}}{\mathcal{C}}} \frac{\omega}{1/\sqrt{\mathcal{L}\mathcal{C}}} l = j\omega \mathcal{L} l$

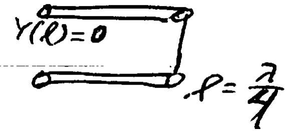
⑤ $l = \frac{\lambda}{4}$ input admittance of shorted stub

$Y(l) = \frac{1}{Z(l)} = \frac{1}{jZ_0 \tan(\beta l)} = -jY_0 \cot(\beta l)$

$l = \frac{\lambda}{4}$, $\beta = \frac{2\pi}{\lambda}$, $\beta l = \frac{\pi}{2}$

$Y(l) = 0$, input becomes an open

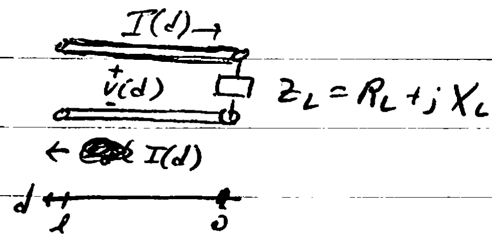
$\mathcal{L}, \mathcal{C}$



1/2 and 1/4 wave transformers

Switch to arbitrary reactive load, $Z_L = R_L + jX_L$

Constrain l to be $n\frac{\lambda}{4}$



$V(d) = V^+ e^{j\beta d} + V^- e^{-j\beta d}$

$I(d) = \left(V^+ e^{j\beta d} - V^- e^{-j\beta d}\right)/Z_0$

Half-wave transformer, $l = \frac{\lambda}{2}$

Look at input $d = l = \frac{\lambda}{2}$

Recall: $\beta = \frac{2\pi}{\lambda}$

$$e^{\pm j\beta\lambda/2} = e^{\pm j\frac{2\pi}{\lambda}\frac{\lambda}{2}} = e^{\pm j\pi} = \cos\pi + j\sin\pi = -1$$

$$\cos(-\pi) + j\sin(-\pi) = -1$$

$$V_{in} \equiv V\left(\frac{\lambda}{2}\right) = -V^+ - V^- = -V(0) = -V_L$$

$$I_{in} \equiv I\left(\frac{\lambda}{2}\right) = \frac{-V^+ + V^-}{Z_0} = -I(0) = -I_L$$

Observations: Inverts sign of V, I at load

input impedance = load $Z_{in} = \frac{V_{in}}{I_{in}} = \frac{-V_L}{-I_L} = Z_L$

Quarter-wave transformer, $l = \frac{\lambda}{4}$

$$e^{\pm j\beta\frac{\lambda}{4}} = e^{\pm j\frac{\pi}{2}} = \begin{cases} \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} \\ \cos\left(-\frac{\pi}{2}\right) + j\sin\left(-\frac{\pi}{2}\right) \end{cases} = \pm j$$

$$V_{in} \equiv V\left(\frac{\lambda}{4}\right) = jV^+ - jV^- = j\,I(0)\,Z_0 \quad \left(\text{recall } I(0) = \frac{V^+ - V^-}{Z_0}\right)$$

$$V_{in} = j\,I_L Z_0$$

$$I_{in} \equiv I\left(\frac{\lambda}{4}\right) = \frac{jV^+ + jV^-}{Z_0} = \frac{j\,V(0)}{Z_0} = j\frac{V_L}{Z_0}$$

Wed 29 Nov Lecture 34

$\frac{\lambda}{4}$ transformer continued, line impedance, generalized reflection coefficient

From last time: $V_{in} \equiv V(\frac{\lambda}{4}) = jV^+ - jV^- = jI(0)Z_0$

$$V_{in} = jI_L Z_0$$

$$I_{in} \equiv I(\tfrac{\lambda}{4}) = \frac{jV(0)}{Z_0} = \frac{jV_L}{Z_0}$$

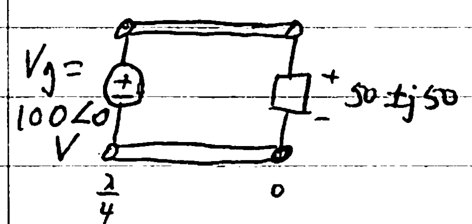
Observations: ① $Z_{in} = \dfrac{V_{in}}{I_{in}} = \dfrac{jI_L Z_0}{jV_L/Z_0} = \dfrac{Z_0^2}{V_L/I_L} = \dfrac{Z_0^2}{Z_L}$

② $I_L = \dfrac{V_{in}}{jZ_0} = \dfrac{-jV_{in}}{Z_0}$ ← load current is independent of Z_L

$$V_L = Z_L \cdot I_L$$

Example | Given $Z_L = 50 + j50\,\Omega$, $\frac{\lambda}{4}$ transformer w/ $Z_0 = 50\,\Omega$

Solve Z_{in}



$$= \frac{Z_0^2}{Z_L} = \frac{50^2}{50+j50} = \frac{50}{1+j} \cdot \frac{1-j}{1-j}$$

$$Z_{in} = 25 - j25\,\Omega$$

With given V_G, what is I_L, avg power absorbed by load?

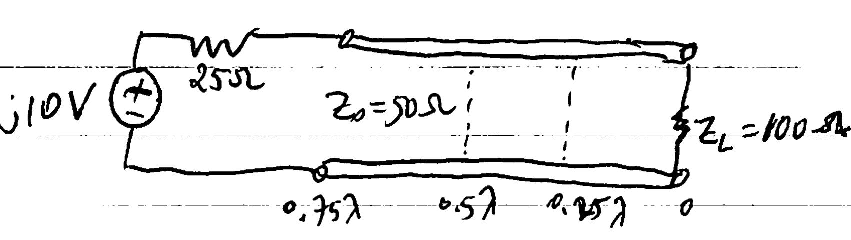
$$I_L = -j\frac{100}{50} = -j2\text{ A},\quad V_L = (50+j50)(-j2) = 100 - j100\text{ V}$$

$$P_L = \frac{1}{2}\mathrm{Re}\{V_L I_L^*\}$$

$$= \frac{1}{2}\mathrm{Re}\{(100 - j100)(j2A)\} = \underline{100\text{ W}}$$

Example | $V_0 = 50\,\Omega$, $Z_L = 100\,\Omega$, $l = 0.75\lambda$, $V_g = j10$, $Z_g = 25\,\Omega$

Solve: V_L, I_L



Approach: Figure out Z_{in} by "going to" $d = 0.5\lambda$, then "advancing" $\lambda/4$ to generator

At $d = 0.5\lambda$, $Z_{in} = Z_L = 100\,\Omega$

Move another $\frac{\lambda}{4}$ and apply quarter-wave equations

$$Z_{in} = \frac{Z_0^2}{Z(0.5\lambda)} = \frac{50^2}{100} = 25\,\Omega$$

$$V_{in} = V_g \frac{Z_{in}}{Z_g + Z_{in}} = \underline{j5V}$$

Next, work in opposite direction

Recall for $\frac{\lambda}{2}$ transformer, $V_{in} = -V(0)$

But here, $V_{in} = -V(\frac{\lambda}{4})$

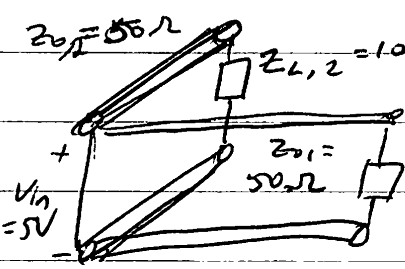
Then use $\frac{\lambda}{4}$ current forcing equation

$$I_L = -j\frac{V_{in}}{Z_0}$$

$$= \frac{-j \cdot -j5V}{50} = \underline{-0.1A}$$

$$V_L = Z_L \cdot I_L = 100 \cdot -0.1 = -\underline{10}V$$

Example Parallel $\frac{\lambda}{4}$ transformers



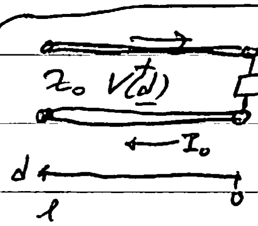
Determine $I_{L,1}$, $I_{L,2}$ and total avg. power absorbed

$$I_{L,1} = I_{L,2} = -j\frac{V_{in}}{Z_{0,1,2}} = -j0.1A$$

$$V_{L,1} = -j5V$$

$$V_{L,2} = -j10V$$

$$P = \tfrac{1}{2}\left[Re\{V_{L,1} I_{L,1}^*\} + Re\{V_{L,2} I_{L,2}^*\}\right] = \underline{0.75\,W}$$



$$I(d) = V^+ e^{j\beta d}(1 - \Gamma_L e^{-2j\beta d})/Z_0$$

$$V(d) = V^+ e^{j\beta d} + V^- e^{-j\beta d}$$

$$I(d) = \left(V^+ e^{j\beta d} - V^- e^{-j\beta d}\right)/Z_0$$

If you cut TL at arbitrary d, replace loaded TL stub w/ some load $\Gamma(d)$, then is no change to $V(d)$, $I(d)$

Develop more general tools to solve V^+, V^-

Start at load: $V(0) = Z_L I(0)$

$d = 0$

$$V^+ + V^- = \frac{Z_L}{Z_0}(V^+ - V^-)$$

$$Z_0 V^- + Z_L V^- = Z_L V^+ - Z_0 V^+$$

$$V^- = \frac{Z_L - Z_0}{Z_L + Z_0} V^+$$

Γ_L load reflection coefficient

$$V(d) = V^+ e^{j\beta d} + \Gamma_L e^{-j\beta d} = V^+ e^{j\beta d}(1 + \Gamma_L e^{-2j\beta d}) \equiv \Gamma_d$$ generalized reflection coefficient

Friday 1 December Lecture 35 Smith Chart

$\Gamma(d) = \Gamma_L e^{-2j\beta d}$ Generalized Reflection coefficient

$V(d) = V^+ e^{j\beta d}(1+\Gamma(d))$ $\quad I(d) = V^+ e^{j\beta d}(1-\Gamma(d))/Z_0$

Line impedance $\quad Z(d) = \dfrac{V(d)}{I(d)} = Z_0 \dfrac{1+\Gamma(d)}{1-\Gamma(d)}$

$Z = Z_d$

$\Gamma = \Gamma(d)$ $\quad \dfrac{Z}{Z_0} = \dfrac{1+\Gamma}{1-\Gamma} \Leftrightarrow \Gamma = \dfrac{Z-Z_0}{Z+Z_0}$

$Z(d) = R(d) + jX(d)$, $\quad \Gamma(d) = \Gamma_L e^{-j2\beta d}$

① For real Z_0 $\quad R(d) \geq 0$, $|\Gamma(d)| \leq 1$

$$|\Gamma| = \frac{|Z-Z_0|}{|Z+Z_0|} = \frac{\sqrt{(R-Z_0)^2+X^2}}{\sqrt{(R+Z_0)^2+X^2}}$$

Mean: 69.8

Median: 71

stdev: 14

Get compass/protractor

Monday 4 December Lecture 36

Mostly Smith chart examples,

① Determine $z(0) = \frac{Z_L}{Z_0} = \frac{50+j100}{50} = 1+j2$

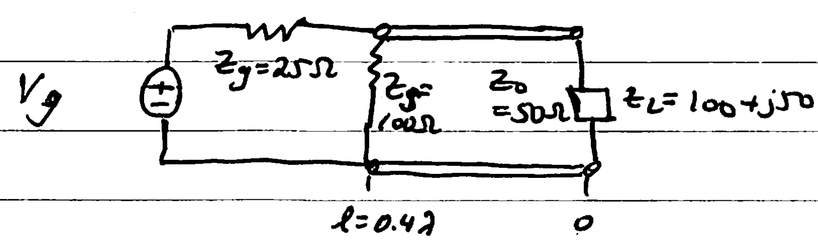
② Locate intersection of resistive/reactive components

Get magnitude of reflection coefficient from bottom of chart

Angle of reflection coefficient

Voltage standing wave coefficient

Lec 35 Ex 2



① $z(0) = \frac{Z_L}{Z_0} = \frac{100+j50}{50} = 2+j1$ ← locate on Smith chart

0.212 wavelengths toward generator

$z(l) = 0.6 + j\,0.65$

$y(l) = 0.75 - j\,0.85$

Normalized shunt impedance $z_s = \frac{100}{50} = 2$

" " admittance $y_s = \frac{1}{2}$

$y_{in} = \frac{1}{2} + \underbrace{0.75 - j\,0.83}_{y(l)} = 1.25 - j\,0.83$

$z_{in} = \frac{1}{y_{in}} = 0.56 + j\,\cancel{0.36}\,0.37$

$Z_{in} = Z_0 z_{in} = 27.8 + j\,18.4\Omega$, $Y_{in} = Y_0\, y_{in} = 0.025 - j0.0175$ S

$V_g = 100 \angle 0$ V, solve avg power to TL, shunt, load.

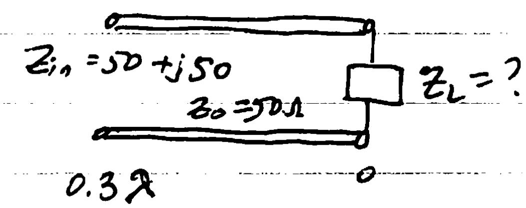
(a) $Z_{in} = 27.8 + j\,18.4\ \Omega$

$$V_{in} = V_g \frac{Z_{in}}{Z_g + Z_{in}}, \quad I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{V_{in}}{Z_{in}}$$

$$P = \frac{1}{2}\mathrm{Re}\{V_{in} I_{in}^*\} = \frac{1}{2}\mathrm{Re}\left\{\frac{V_g \cdot Z_{in}}{Z_g + Z_{in}} \cdot \left(\frac{V_g}{Z_g + Z_{in}}\right)^*\right\} = 44.4\ W$$

(b) $P_g = \frac{1}{2}\mathrm{Re}\left\{V_{in}\left(\frac{V_{in}}{Z_g}\right)^*\right\} = \frac{V_{in}^2}{2Z_g} = 17.8\ W$, rest to load.

Example 3, Lecture 35



$$z_{in} = \frac{Z_{in}}{Z_0} = \frac{50 + j50}{50} = 1 + j$$

~ 0.162

$z(0) \approx 0.78 - 0.8j$

Textbook: $z(0) \approx 0.76 - j\,0.84$

$Z_L = Z_0 \cdot z(0)$, $Y_L = \frac{1}{50} \cdot y(0)$

$d = 0.3\lambda$

CCW 0.3λ from $0.162 \rightarrow 0.138$

Unknown load z_L on $Z_0 = 50\Omega$ TL

$V(d_{min}) = 20$ V, $d_{min} = 0.125\lambda$, VSWR = 4

Determine Z_L, P_L

WED DEC 6 LEC

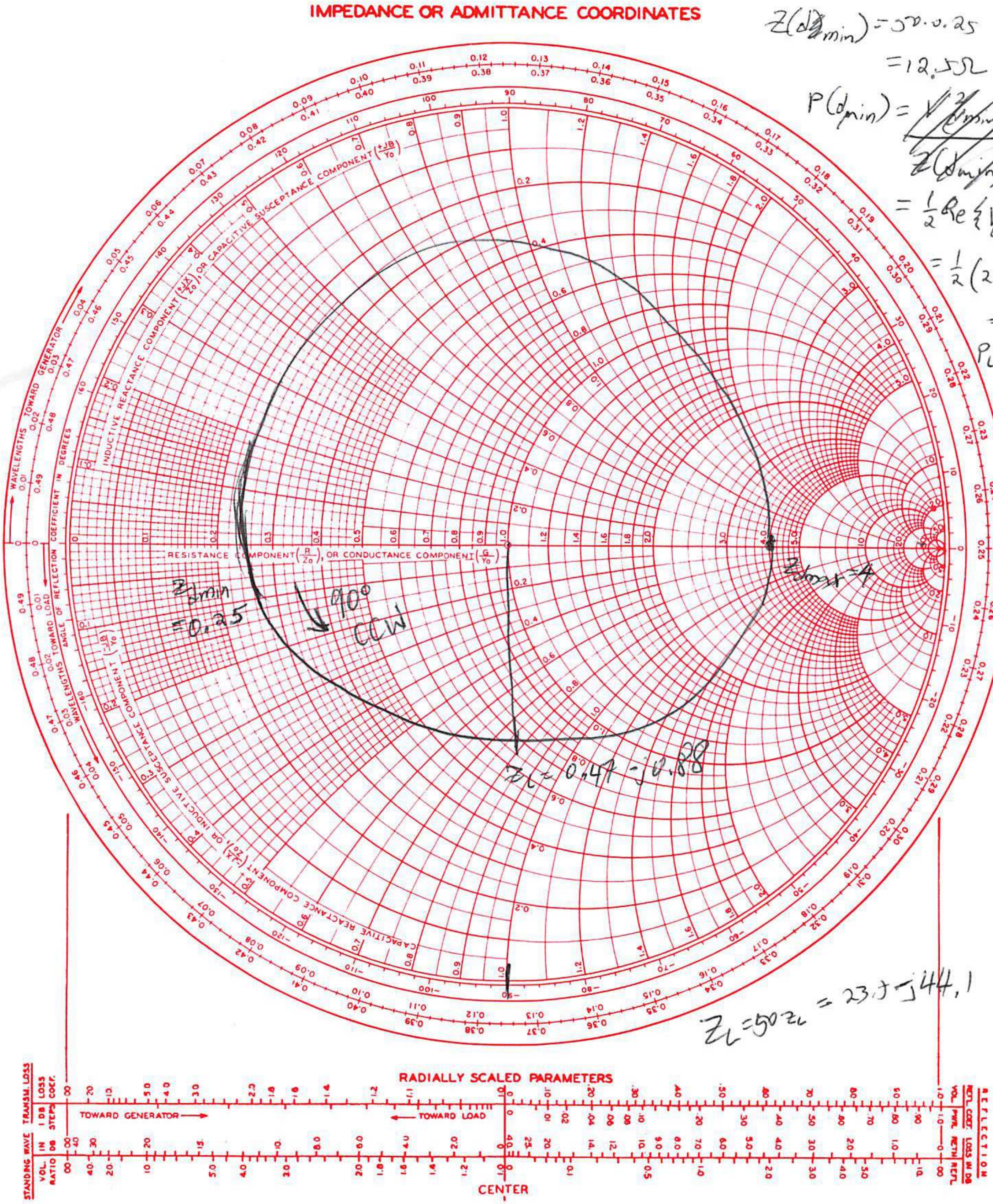
CCW clock

$z_{d_{min}} = 0.25$

$Z(d_{min}) = 50 \cdot 0.25 = 12.5\Omega$

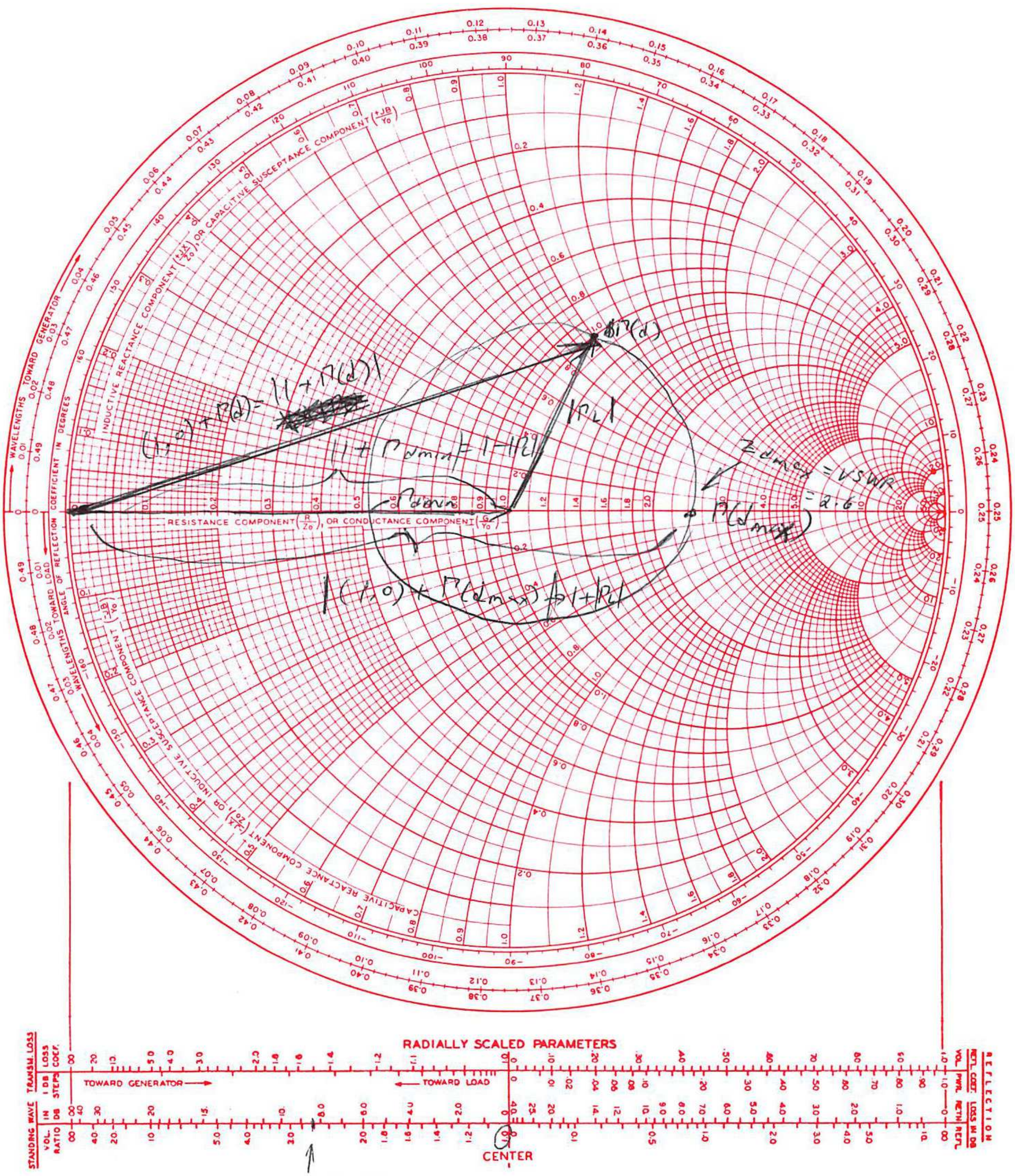
$P(d_{min}) = \cancel{\frac{V^2(d_{min})}{Z(d_{min})}} = \frac{1}{2} Re\{V_{d_{min}} I^*_{d_{min}}\} = \frac{1}{2}\left(20 \cdot \frac{20}{12.5}\right) = 16\ W$

$P_L = P_{d_{min}} = 16\ W$



$Z_L = 50 z_L = 23.5 - j44.1$

IMPEDANCE OR ADMITTANCE COORDINATES



VSWR = 2.7

If $|\Gamma_L| = 0$, VSWR = 1 ←

If $|\Gamma_L| = 1$, VSWR = ∞ ← short, open, pure reactive load (on periphery of unit circle)

Wed 6 December lecture 37 (lec. 36 in course notes)

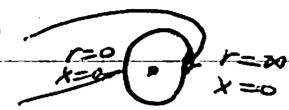
$V(d) = V^+ e^{j\beta d}(1 + \Gamma_L e^{-2j\beta d})$, $I(d) = \dfrac{V^+ e^{j\beta d}(1-\Gamma_L e^{-2j\beta d})}{Z_0}$

Piston animation fr. Wikipedia

Unless $\Gamma_L = 0$ (recall $\Gamma_L \equiv \frac{Z_L - Z_0}{Z_L + Z_0}$), impedance matched case, there will be a reflected ~~coefficient~~ component.

Pure standing waves open circuit $\Gamma_L = 1$

short circuit $\Gamma_L = -1$



$|V(d)| = |V^+|\,|1+\Gamma(d)|$

$|V(d)|_{max} = |V^+|(1+|\Gamma_L|)$ and $|V(d)|_{max} = |V^+|(1-|\Gamma_L|)$

at $d = d_{max}$ at $d = d_{min}$

$d_{max} - d_{min}$ is odd multiple of $\frac{\lambda}{4}$ (180° around Smith Chart)

Voltage Standing Wave Ratio (VSWR) $= \dfrac{|V(d_{max})|}{|V(d_{min})|} = \dfrac{1+|\Gamma_L|}{1-|\Gamma_L|}$

$\Updownarrow$

$z = \dfrac{1+\Gamma}{1-\Gamma} \Leftrightarrow \Gamma = \dfrac{z-1}{z+1} \leftarrow$ another bilinear transformation $\longrightarrow |\Gamma_L| = \dfrac{VSWR-1}{VSWR+1}$

$\Gamma(d_{min}) = |\Gamma_L|$, $VSWR = \dfrac{1+\Gamma_{dmax}}{1-\Gamma_{dmin}}$

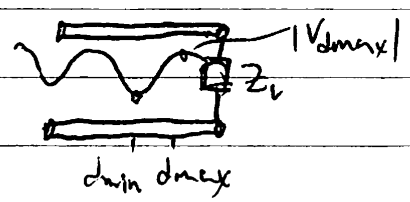
$z(d_{max}) = VSWR$ read directly off SC

If you have TL and unknown Lab $\Rightarrow$ can measure VSWR

Suppose d_{max} is known d_{max}, d_{min} $\Downarrow$ $|\Gamma_L|$

$\Gamma(d_{max}) = \Gamma_L e^{-j2\beta d_{max}} = |\Gamma_L| \leftarrow$ don't know Γ_L

$Z_L = \dfrac{1+\Gamma_L}{1-\Gamma_L}$ $\overline{\text{OR}}$ go to $\Gamma(d_{max})$, rotate CCW



Friday 8 December Lecture 38 (lec 37 notes)

Impedance matching

For lossless TL, power input at generator end, P_{in}, matches power delivered to load, P_L, also matching $P(d)$ at any d,

$$P(d) = \frac{1}{2}\mathrm{Re}\{V(d)I^*(d)\}$$

$$= \frac{1}{2}\mathrm{Re}\left\{\left(V^+e^{j\beta d} + V^-e^{-j\beta d}\right)\left(\frac{V^+e^{j\beta d} - V^-e^{-j\beta d}}{Z_0}\right)^*\right\}$$

$$= \underbrace{\frac{|V^+|^2}{2Z_0}}_{\text{forward going wave}} - \underbrace{\frac{|V^-|^2}{2Z_0}}_{\text{reflected wave}}$$

Recall L34:

$$V^- = \underbrace{\frac{Z_L - Z_0}{Z_L + Z_0}}_{\Gamma_L} V^+$$

$$P(d) = \frac{|V^+|^2}{2Z_L}\left(1 - |\Gamma_L|^2\right)$$

$|\Gamma_L|^2$ power reflection coefficient

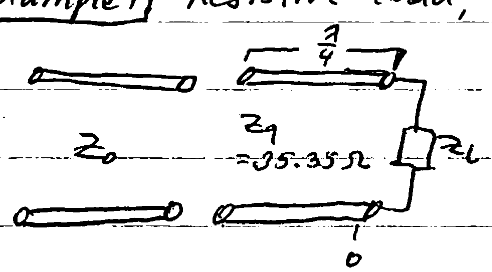
If $Z_L \neq Z_0$, then there will be reflected power and VSWR > 1

Aside: 13.56 MHz RF plasma tools

Load is a plasma, matching network answers good power transfer

In general $Z_L \neq Z_0$

Example 1| Resistive load, $\frac{\lambda}{4}$ wave TL to match



$Z_L = 25\,\Omega$

$R_g = Z_0 = 50\,\Omega$

Insert $\frac{\lambda}{4}$ transformer right next to load, $d = 0$

$Z_q = Z_0$ of $\frac{\lambda}{4}$ transformer

$Z_q = \sqrt{Z_L \cdot Z_{in}}$

$= 35.35\,\Omega$

Recall L33:

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

$\frac{\lambda}{4}$ input impedance

FRI 8 DEC Lec

Smith Chart 1

① Locate normalized load on SC = 1+j1

② Rotate to pure real z

③ Note z_{dmax} = VSWR = 2.6 at $d_{max} = 0.088\lambda$

④ $Z(d_1) = 131\,\Omega$

$Z_q = \sqrt{131 \cdot 50}$

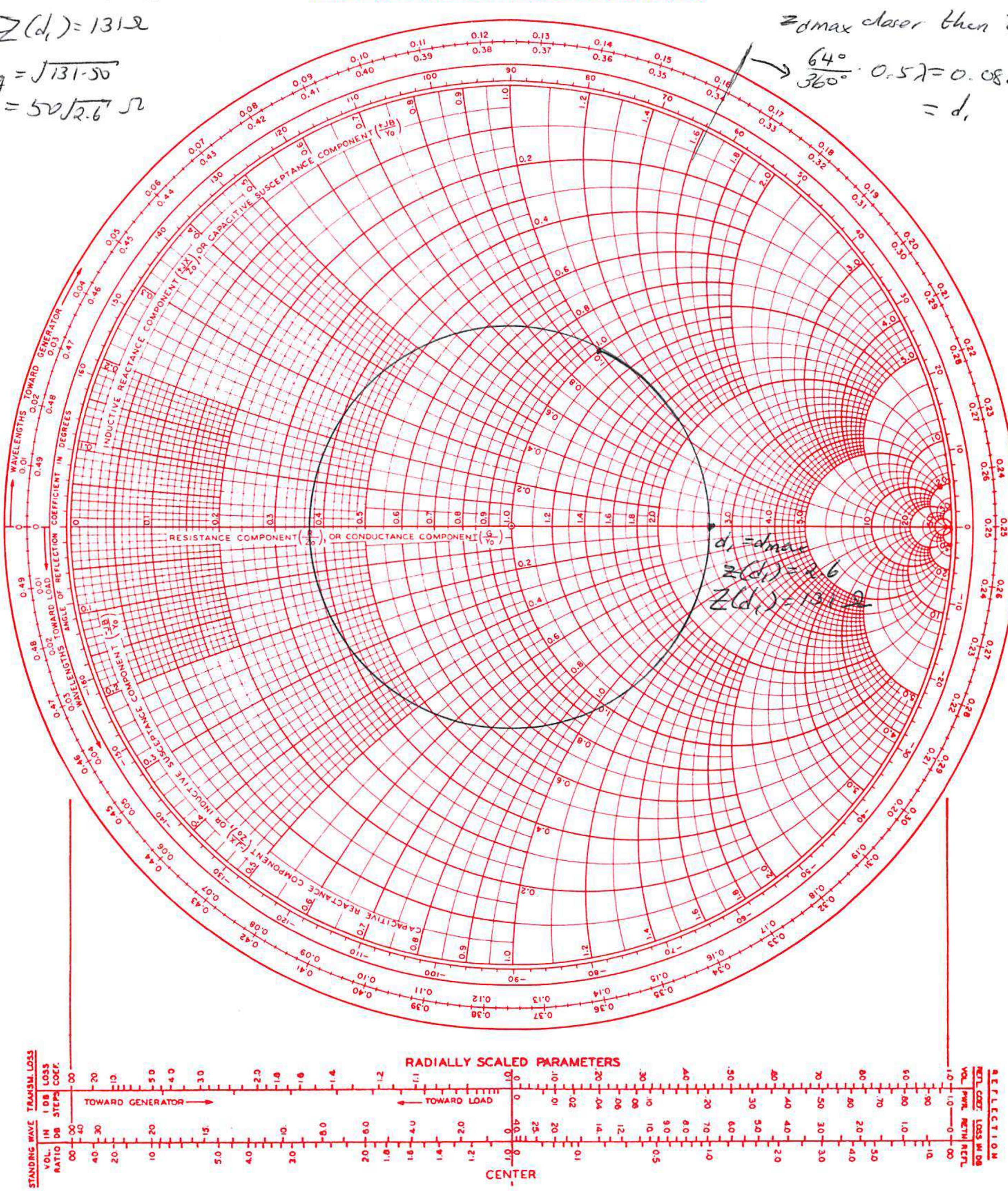
$= 50\sqrt{2.6}\ \Omega$

z_{dmax} closer than z_{dmin}

$\frac{64°}{360°} \cdot 0.5\lambda = 0.088\lambda$

$= d_1$

IMPEDANCE OR ADMITTANCE COORDINATES



$z_L = \frac{Z_L}{Z_0} = 2 - j1$

Smith Chart 2

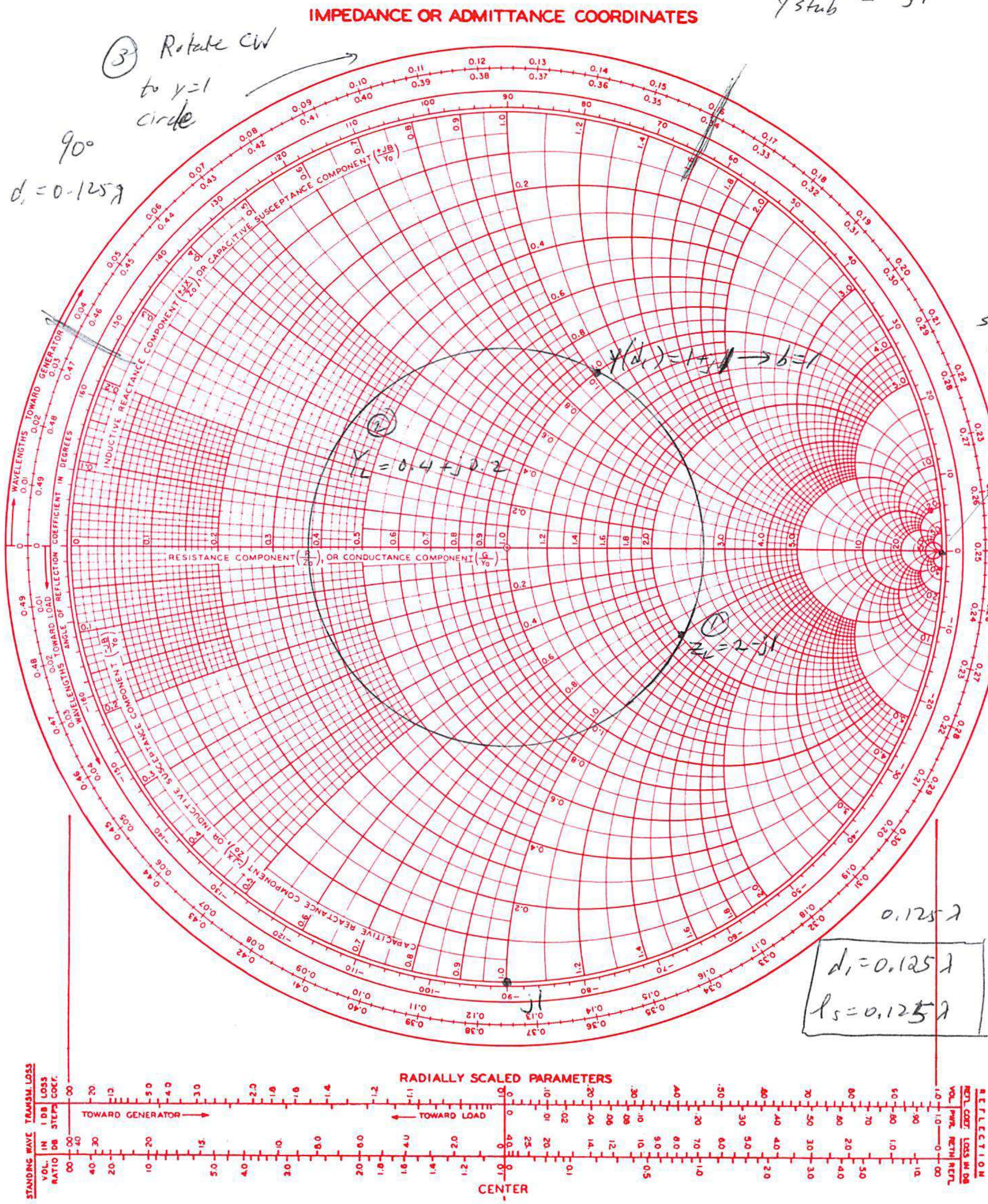
④ Want $y(d_1) + y_{stub} = 1 + j0$

$y_{stub} = -j1$

③ Rotate CW to $y=1$ circle

90°

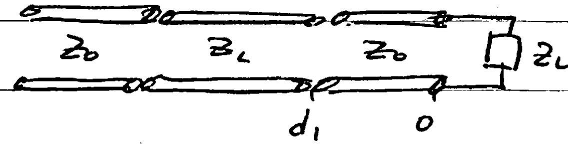
$d_1 = 0.125\lambda$



0.125λ

$\boxed{\begin{array}{l} d_1 = 0.125\lambda \\ l_s = 0.125\lambda \end{array}}$

Example 2 $\frac{\lambda}{4}$ match to resistive load



$Z_L = 50 + j50$, $R_g = Z_0 = 50\,\Omega$

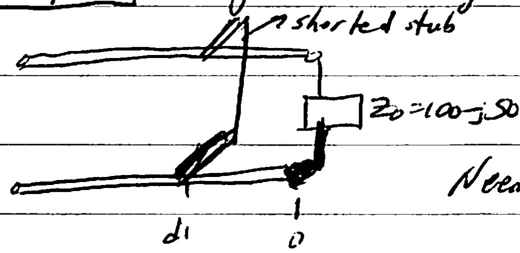
Want a purely resistive $Z(d_1)$ such that $Z_q = \sqrt{Z(d_1) \cdot 50}$ is also purely real

Where on Smith Chart / $|\Gamma_L|$ circle is $Z(d)$ purely real?

Either d_{max} or d_{min} (i.e., on x-axis)

(See Smith Chart 1)

Example 3 Single-stub tuning $\qquad R_g = Z_0 = 50\,\Omega$



Need to determine z_1 and l_s

Want: $y(d_1) = 1 + jb$ ← a constant to determine

and

$y_{stub} = -jb$ input admittance

perfect impedance match

Combined normalized ~~input~~ admittance $y(d_1) + y_{stub} = 1 + j0$

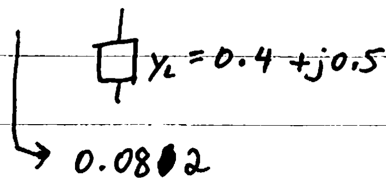
ECE 329

Mon 11 Dec Lecture

Lee
Sec X

① Step 1

Locate 0.4 + j0.5, note its angle/position from SC



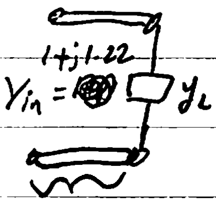
② Step 2

Rotate to $y(d_1) = 1 + jb$ r=1 circle, g=1

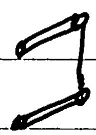
read off 'b' from SC

$b = 1.22$, $y(d_1) = 1 + j1.22$

read off $d_1 = 0.086\lambda$



③ Find l_s by cancelling imaginary part of $y(d_1)$



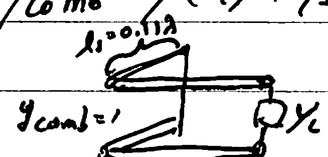
- Identify shorted stub w/ $Y_{in} = -j1.22$
- Start on center ~~left~~ right edge of SC $r=0 \rightarrow y=\infty$
- Rotate on unit circle to $-j1.22$
- Read off angle/distance

(outer edge of SC) → purely reactive case

$\sim -78°$ $l_s = 0.109\lambda$

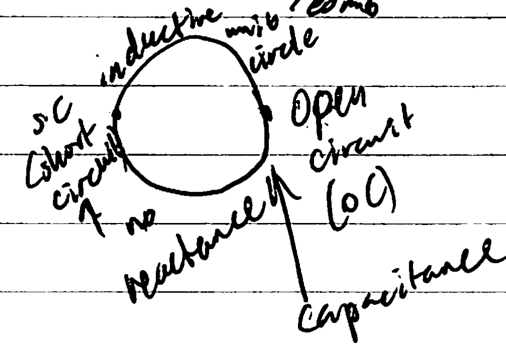
④ Final answer

$y_{comb} = y(d_1) + y_{ss} = 1$



$Z_{in} = Z_0 z_{in}$

$Y_{comb} = Y_0 \cdot y_{comb}$



What is l_s of open stub?

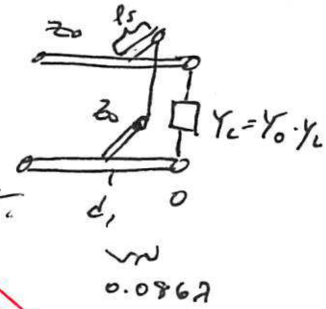
$$\boxed{\begin{array}{l} l_s = 0.36\lambda \\ \text{open} \end{array}}$$

$180° \times \frac{\lambda}{4}$.

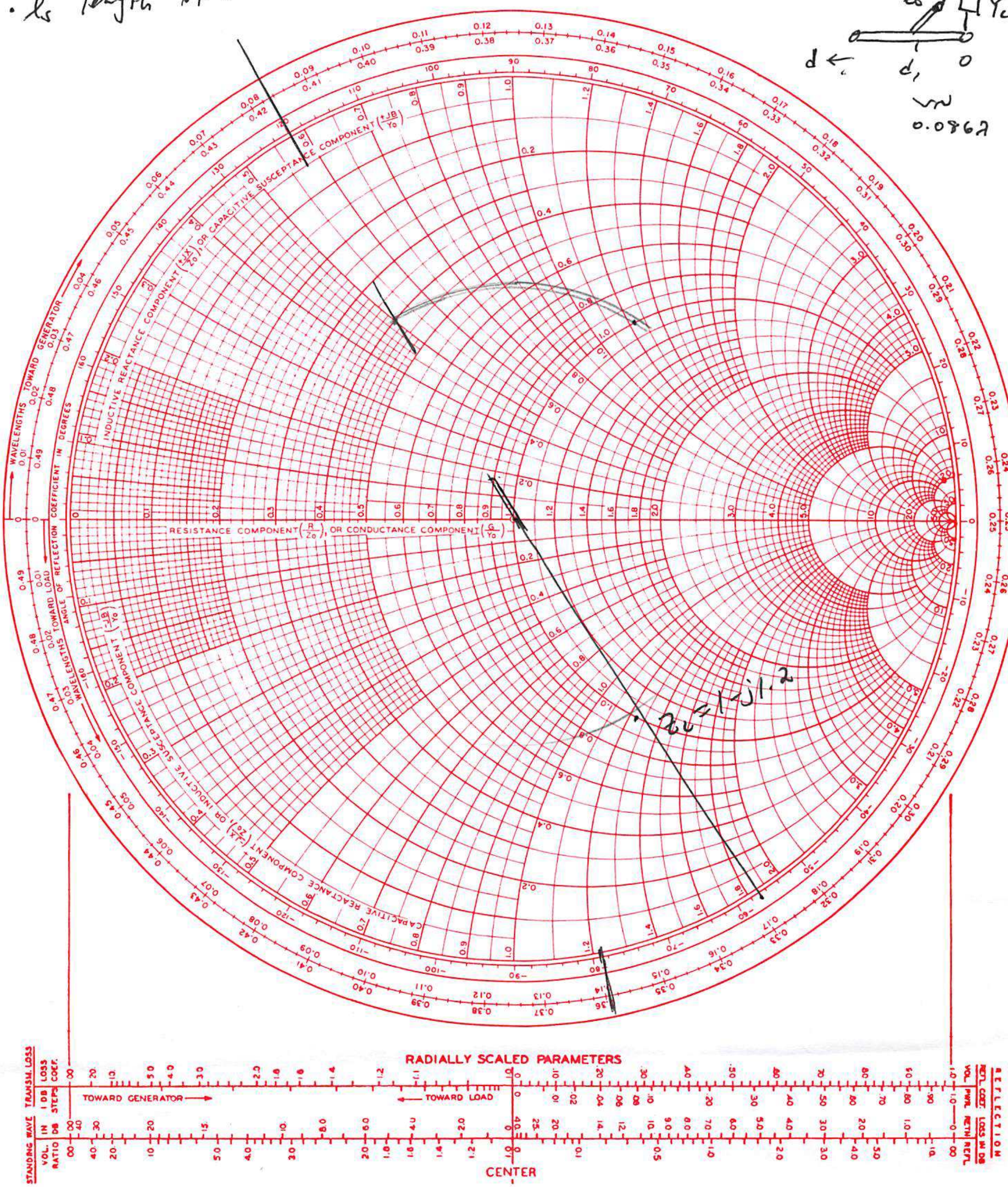
Mon 11 Dec lecture Practice Problem

Example| use shorted single stub to impedance match $y_L = 0.4 + j0.5$

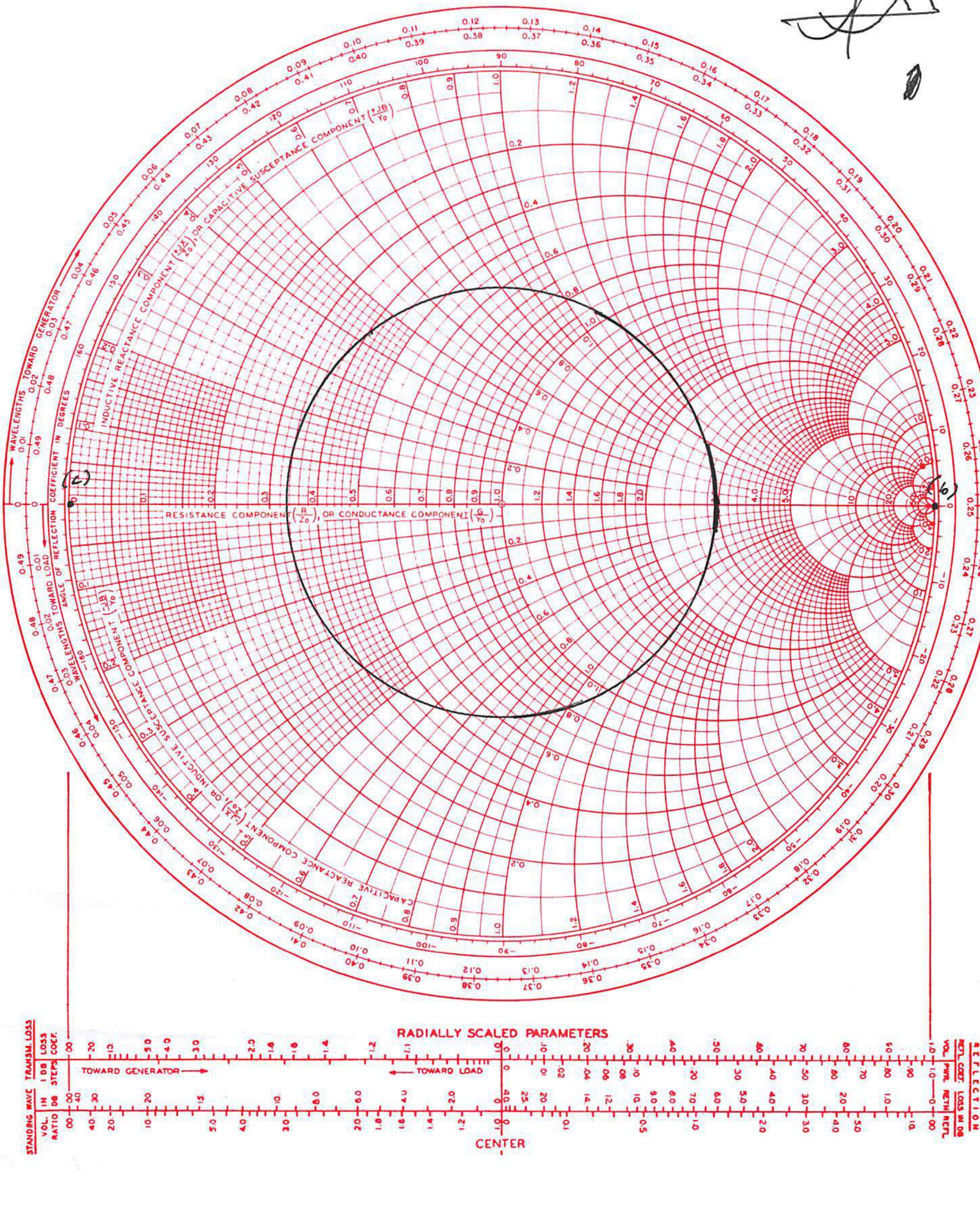
- d_1 stub insertion location
- l_s length of shorted stub

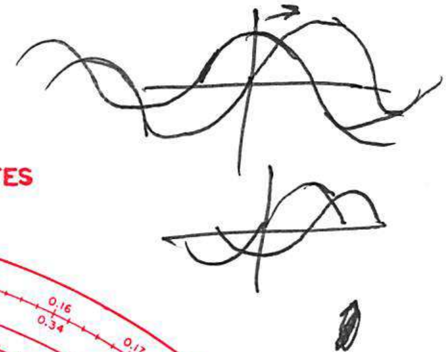


IMPEDANCE OR ADMITTANCE COORDINATES



IMPEDANCE OR ADMITTANCE COORDINATES





bilinear transformation

6-8 Q's, TA review, Q professor review docs.

Pratik ECE 329 Thurs 14 Dec Review session

Smith Chart: Quarter wave transformer

Smith Chart: Calculate Z_L given VSWR

1. Calc d_{min} & d_{max}

Smith Chart: Impedance matching using stubs

Shuo Gauss' law / Maxwell's eqns

$$\iint \vec{J} \, d\vec{S} = \iiint \underbrace{\frac{d\rho}{dt}}_{>0} dV$$

$$\hat{n} \times (\vec{H}_+ - \vec{H}_-) = \vec{J}$$

b) $\langle S_i \rangle = \dfrac{|E_0|}{2\eta_1}$

1. $\Gamma_{in} = \Gamma_L e^{-2j} \cdots$

$= j e^{-j2 \frac{2\pi}{\lambda} \cdot \frac{1}{8}\lambda}$

$= j e^{-j\frac{\pi}{2}} = j \ldots$

$Z_{in} = \infty$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = 1$$

$$\Gamma_L = \frac{2Z_0 - Z_0}{2Z_0 + Z_0} = \frac{1}{3}$$

$$P_{L,avg} = \frac{|V^+|^2}{2Z_0} - \frac{|V^-|^2}{2Z_0}$$

$V^- = V^+ \sqrt{2} \rightarrow \frac{V^-}{\Gamma_L}$

$V(d) = V^+ e^{j\beta d} + V^- e^{-j\beta d}$

$= V^-\left(\frac{1}{\sqrt{2}} e^{j\beta d} + e^{-j\beta d}\right) = V^- e^{-j\beta d}\left(1 + \frac{1}{\sqrt{2}} e^{j2\beta d}\right)$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$Z_L = jx + 0$

$$|\Gamma_L| = \left|\frac{jx-1}{jx+1}\right| = \left|\frac{\sqrt{1+x^2}\, e \cdots}{\sqrt{1+x^2}\, e \cdots}\right|$$